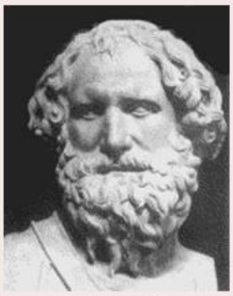
The

Greatest Mathematicians of All Time


Isaac Newton


Carl Gauss


Archimedes


Leonhard Euler


Euclid


Bernhard Riemann


Henri Poincaré


David Hilbert


Alex. Grothendieck


J.-L. Lagrange


G.W. Leibniz


Pierre de Fermat

The Greatest Mathematicians of All Time

ranked in approximate order of "greatness."

To qualify, the mathematician's work must have **breadth**, **depth**, and **historical importance.**

1. Isaac Newton
2. Carl F. Gauss
3. Archimedes
4. Leonhard Euler
5. Euclid
6. Bernhard Riemann
7. Henri Poincaré
8. David Hilbert
9. Alexander Grothendieck
10. Joseph-Louis Lagrange
11. Gottfried W. Leibniz
12. Pierre de Fermat
13. Niels Abel
14. Évariste Galois
15. John von Neumann

16.	Srinivasa Ramanujan	21.	Augustin Cauchy	26.	Arthur Cayley
17.	Karl W. T. Weierstrass	22.	Leonardo `Fibonacci'	27.	Amalie Emma Noether
18.	Brahmagupta	23.	Georg Cantor	28.	Gustav Lejeune Dirichlet
19.	René Déscartes	24.	Hermann K. H. Weyl	29.	Kurt Gödel
20.	Eudoxus of Cnidus	25.	Carl G. J. Jacobi	30.	Pythagoras of Samos

At some point a longer list will become a List of Great Mathematicians rather than a List of Great*est* Mathematicians. I've expanded the List to Sixty, but you may prefer to leave it at Forty or Thirty or even prune it back to just a Top Twenty or Top Fifteen or Top Ten List.

31.	Bháscara Áchárya	36.	Pierre-Simon Laplace	41.	Stefan Banach
32.	Blaise Pascal	37.	Charles Hermite	42.	Gaspard Monge
33.	Apollonius of Perga	38.	André Weil	43.	George Boole
34.	William Rowan Hamilton	39.	Richard Dedekind	44.	Takakazu Seki
35.	Muhammed al-Khowârizmi	40.	Felix Christian Klein	45.	L.E.J. Brouwer

46.	François Viète	51.	Johannes Kepler	56.	Albert Einstein
47.	Ferdinand Eisenstein	52.	Alhazen ibn al-Haytham	57.	Jean-Victor Poncelet
48.	Christiaan Huygens	53.	Jacques Hadamard	58.	Andrey N. Kolmogorov
49.	Pappus of Alexandria	54.	Omar al-Khayyám	59.	Liu Hui
50.	Girolamo Cardano	55.	Hipparchus of Nicaea	60.	Jacob Bernoulli

---

Earliest mathematicians

Little is known of the earliest mathematics, but the famous *Ishango Bone* from Early Stone-Age Africa has tally marks suggesting arithmetic. The markings include six prime numbers (5, 7, 11, 13, 17, 19) in order, though this is probably coincidence.

The advanced artifacts of Egypt's Old Kingdom and the Indus-Harrapa civilization imply strong mathematical skill, but the first written evidence of advanced arithmetic dates from Sumeria, where 4500-year old clay tablets show multiplication and division problems; the first abacus may be about this old. By 3600 years ago, Mesopotamian tablets show tables of squares, cubes, reciprocals, and even logarithms, using a primitive place-value system (in base 60, not 10). Babylonians were familiar with the Pythagorean theorem, quadratic equations and even cubic equations (though they didn't have a general solution for these).

Also at least 3600 years ago, the Egyptian scribe Ahmes produced a famous manuscript (now called the *Rhind Papyrus*), itself a copy of a late Middle Kingdom text. It showed simple algebra methods and included a table giving optimal expressions using Egyptian fractions. (Today, Egyptian fractions lead to challenging number theory problems with no practical applications, but they may have had practical value for the Egyptians. To divide 17 grain bushels among 21 workers, the equation 17/21 = 1/2 + 1/6 + 1/7 has practical value, especially when compared with the "greedy" decomposition 17/21 = 1/2 + 1/4 + 1/17 + 1/1428.)

While Egyptians may have had more advanced geometry, Babylon was more advanced at arithmetic and algebra. This was probably due, at least in part, to their place-value system. But although their base-60 system survives (e.g. in the division of hours and degrees into minutes and seconds) the Babylonian notation, which used the equivalent of IIIIII XXXXXIIIIIII XXXXIII to denote 417+43/60, was unwieldy compared to the "ten digits of the Hindus."

The Egyptians used the approximation $\pi \approx (4/3)^4$ (derived from the idea that a circle of diameter 9 has about the same area as a square of side 8). The Babylonians used the slightly inferior $\pi \approx 25/8$, perhaps surprising given that they had an approximation for $\sqrt{2}$ better than, and 900 years before, Apastamba's.

Early Vedic mathematicians

The greatest mathematics before the Golden Age of Greece was in India's early Vedic (Hindu) civilization. The Vedics understood relationships between geometry and arithmetic, developed astronomy, astrology, calendars, and used mathematical forms in some religious rituals. (Ancient China also developed mathematics, but little written evidence survives prior to Chang Tshang's famous book.)

The earliest mathematician to whom definite teachings can be ascribed was Lagadha, who apparently lived about 1300 BC and used geometry and elementary trigonometry for his astronomy. Baudhayana lived about 800 BC and also wrote on algebra and geometry; Yajnavalkya lived about the same time and is credited with the then-best approximation to π. Another famous early Vedic mathematician was Apastamba, who lived slightly before Pythagoras, did work in geometry, advanced arithmetic, and may have proved the Pythagorean Theorem. (Apastamba used an excellent approximation for the square root of 2 (577/408, one of the continued fraction *approximants*); a 20th-century scholar has "reverse-engineered" a plausible geometric construction that led to this approximation.) Other early Vedic mathematicians solved quadratic and simultaneous equations.

Thales of Miletus (ca 625 - 546 BC) Greece

Thales was the Chief of the *Seven Sages* of ancient Greece, and he is often called the "Father of Science" or the "First Philosopher." Thales is believed to have studied mathematics under Egyptians, who in turn were aware of much older mathematics from Mesopotamia. Thales may have invented the notion of compass-and-straightedge construction. Several fundamental theorems about triangles are attributed to Thales, including the law of similar triangles (which Thales used famously to calculate the height of the Great Pyramid) and the fact that any angle inscribed in a semicircle is a right angle. (Thales proved this latter fact using two of his other theorems. It is said that Thales sacrificed an ox to celebrate what might have been the very first mathematical proof!)

Thales was also an astronomer; he invented the 365-day calendar and is the first person believed to have correctly predicted a solar eclipse. His theories of physics would seem quaint today, but he seems to have been the first to describe magnetism and static electricity. Thales was also a famous politician, ethicist, and military strategist. Aristotle said, "To Thales the primary question was not what do we know, but how do we know it." Thales' writings have not survived and are known only second-hand. Since his famous theorems of geometry were probably already known in ancient Babylon, his importance derives from imparting the notions of mathematical proof and the scientific method to ancient Greeks.

Thales' student and successor was Anaximander, who is often called the "First Scientist" instead of Thales: his theories were more frmly based on experimentation and logic, while Thales still relied on some animistic interpretations. Anaximander is famous for astronomy, cartography and sundials, and also enunciated a theory of evolution, that land species somehow developed from primordial fish! Anaximander's most famous student, in turn, was Pythagoras.

Pythagoras of Samos (ca 578-505 BC) Greece

Pythagoras, who is sometimes called the "First Philosopher," studied under Anaximander, Egyptians, Babylonians, and the mystic Pherekydes (from whom Pythagoras acquired a belief in reincarnation); he became the most influential of early Greek mathematicians. He is credited with being first to use axioms and deductive proofs, so his influence on Plato and Euclid may be enormous. He and his students (the "Pythagoreans") were ascetic mystics for whom mathematics was partly a spiritual tool. (Some occultists treat Pythagoras as a wizard and founding mystic philosopher.) Pythagoras was very interested in astronomy and recognized that the Earth was a globe similar to the other planets. He believed thinking was located in the brain rather than heart.

Despite Pythagoras' historical importance I may have ranked him too high: many results of the Pythagoreans were due to his students; none of their writings survive; and what is known is reported second-hand, and possibly exaggerated, by Plato and others. His students included Hippasus of Metapontum, perhaps the famous physician Alcmaeon, Milo of Croton, and Croton's daughter Theano (who may have been Pythagoras's wife). The term "Pythagorean" was also adopted by many disciples who lived later; these disciples include Philolaus of Croton, the natural philosopher Empedocles, and several other famous Greeks. Pythagoras' successor was apparently Theano herself: the Pythagoreans were one of the few ancient schools to practice gender equality.

Pythagoras discovered that harmonious intervals in music are based on simple rational numbers. This led to a fascination with integers and mystic numerology; he is sometimes called the "Father of Numbers" and once said "Number rules the universe." (About the mathematical basis of music, Leibniz later wrote, "Music is the pleasure the human soul experiences from counting without being aware that it is counting." Other mathematicians who investigated the arithmetic of music included Huygens, Euler and Simon Stevin.)

The Pythagorean Theorem was known long before Pythagoras, but he is often credited with the first *proof*. (But Apastamba proved it in India at about the same time, and some historians believe Pythagoras journeyed to India and learned of the proof there.) He also discovered the simple parametric form of Pythagorean triplets (xx-yy, 2xy, xx+yy). Other discoveries of the Pythagorean school include the concepts of perfect and amicable numbers, polygonal numbers, golden ratio (attributed to Theano), the five regular solids (attributed to Pythagoras himself), and irrational numbers (attributed to Hippasus). It is said that the discovery of irrational numbers upset the Pythagoreans so much they tossed Hippasus into the ocean!

The famous successors of Thales and Pythagoras included Parmenides of Elea (ca 515-440 BC), Zeno of Elea (ca 495-435 BC), Plato of Athens (ca 428-348 BC), Theaetetus (ca 414-369 BC), and Archytas (friend of Plato, inventor of pulley). These early Greeks ushered in a Golden Age of Mathematics and Philosophy unequaled in Europe until the Renaissance. The emphasis was on pure, rather than practical, mathematics. Plato (who ranks #40 on Michael Hart's famous list of the Most Influential Persons in History) decreed that his scholars should do geometric construction solely with compass and straight-edge rather than with "carpenter's tools" like rulers and protractors.

Eudoxus of Cnidus (408-355 BC) Asia Minor, Greece

Eudoxus journeyed widely for his education, despite that he was not wealthy, studying mathematics with Archytas in Tarentum, medicine with Philiston in Sicily, philosophy with Plato in Athens, continuing his mathematics study in Egypt, touring the Eastern Mediterranean with his own students and finally returned to Cnidus where he established himself as astronomer and physician. What is known of him is second-hand, through the writings of Euclid and others, but he seems to have been one of the great mathematicians of the ancient world.

Many of the theorems in Euclid's *Elements* were first proved by Eudoxus. While Pythagoras had been horrified by the discovery of irrational numbers, Eudoxus is famous for incorporating them into arithmetic. He also developed the earliest techniques of the infinitesimal calculus. Eudoxus (or Pythagoras?) was the first person known to have recognized that the Earth rotates around the Sun.

Four of Eudoxus' most famous discoveries were the volume of a cone, extension of arithmetic to the irrationals, summing formula for geometric series, and viewing π as the limit of polygonal perimeters. None of these seems difficult today, but it does seem remarkable that they were all first achieved by the same man. Eudoxus has been quoted as saying "Willingly would I burn to death like Phaeton, were this the price for reaching the sun and learning its shape, its size and its substance."

Euclid of Megara & Alexandria (ca 322-275 BC) Greece/Egypt

Euclid may have been a student of Aristotle. He founded the school of mathematics at the great university of Alexandria. He was the first to prove that there are infinitely many prime numbers; he proved the unique factorization theorem ("Fundamental Theorem of Arithmetic"); and devised *Euclid's algorithm* for computing gcd. He introduced the Mersenne primes and observed that $(M^2+M)/2$ is always perfect (in the sense of Pythagoras) if **M** is Mersenne. (The converse, that any even perfect number has such a corresponding Mersenne prime, was tackled by Alhazen and proven by Euler.) Among several books attributed to him are *The Division of the Scale* (a mathematical discussion of music), *The Optics*, *The Cartoptrics* (a treatise on the theory of mirrors), a book on spherical geometry, a book on logic fallacies, and his comprehensive math textbook *The Elements*. Several of his masterpieces have been lost, including works on conic sections and other advanced geometric topics. Apparently Desargues' Homology Theorem (a pair of triangles is coaxial if and only if it is copolar) was proved in one of these lost works; this is the fundamental theorem which initiated the study of projective geometry. Euclid ranks #14 on

Michael Hart's famous list of the Most Influential Persons in History. *The Elements* introduced the notions of axiom and theorem; was used as a textbook for 2000 years; and in fact is still the basis for high school geometry, making Euclid the leading mathematics teacher of all time. Some think his best inspiration was recognizing that the Parallel Postulate must be an axiom rather than a theorem.

There are many famous quotations about Euclid and his books. Abraham Lincoln abandoned his law studies when he didn't know what "demonstrate" meant and "went home to my father's house [to read Euclid], and stayed there till I could give any proposition in the six books of Euclid at sight. I then found out what demonstrate means, and went back to my law studies."

Archimedes of Syracuse (287-212 BC) Greece/Sicily

Archimedes is universally acknowledged to be the greatest of ancient mathematicians. He studied at Euclid's school (probably after Euclid's death), but his work far surpassed the works of Euclid. His achievements are particularly impressive given the lack of good mathematical notation in his day. His proofs are noted not only for brilliance but for their "awesome" clarity. Archimedes made advances in number theory, algebra, and analysis, but his greatest contributions were in geometry. He found a method to trisect an arbitrary angle (using a *markable* straightedge the construction is impossible using strictly Platonic rules). One of his most remarkable and famous geometric results was determining the area of a parabolic section, for which he offered two independent proofs, one using his *Principle of the Lever*, the other using a geometric series.

Archimedes' methods anticipated both the integral and differential calculus. His original achievements in physics include the principles of leverage, the first law of hydrostatics, and inventions like the compound pulley, the hydraulic screw, and war machines. His books include *Floating Bodies*, *Spirals*, *The Sand Reckoner*, *Measurement of the Circle*, and *Sphere and Cylinder*. Archimedes proved that the volume of a sphere is two-thirds the volume of a circumscribing cylinder. He requested that a representation of such a sphere and cylinder be inscribed on his tomb.

Recently, modern technology has led to the discovery of new writings by Archimedes, hitherto hidden on a palimpsest, including a note that implies an understanding of the distinction between countable and uncountable infinities (a distinction which wasn't resolved until Georg Cantor, who lived 2300 years after the time of Archimedes). Although Archimedes was certainly one of the greatest genius ever, many listmakers would rank him lower than I have: He was simply *too* far ahead of his time to have great historical significance.

Archimedes discovered formulae for the volume and surface area of a sphere, and may even have been first to notice and prove the simple relationship between a circle's circumference and area. For these reasons, **&pi** is often called *Archimedes' constant*. His approximation **$223/71 < \pi < 22/7$** was the best of his day, though Apollonius soon surpassed it.

Apollonius of Perga (262-190 BC) Greece

Apollonius, called "The Great Geometer," is sometimes considered the second greatest of ancient Greek mathematicians (Euclid and Eudoxus are the other candidates for this honor). His writings on conic sections have been studied until modern times; he invented the names for parabola, hyperbola and ellipse; he developed methods for normals and curvature. Although astronomers eventually concluded it was not physically correct, Apollonius developed the "epicycle and deferent" model of planetary orbits, and proved important theorems in this area. He deliberately emphasized the beauty of pure, rather than applied, mathematics, saying his theorems were "worthy of acceptance for the sake of the demonstrations themselves."

Since many of his works have survived only in a fragmentary form, several great Renaissance and Modern mathematicians (including Vieta, Fermat, Pascal and Gauss) have enjoyed reconstructing and reproving his "lost" theorems. (Among these, the most famous is to construct a circle tangent to three other circles.)

In evaluating the genius of the ancient Greeks, it is well to remember that their achievements were made without the convenience of modern notation. It is clear from his writing that Apollonius almost developed the analytic geometry of Descartes, but failed due to the lack of such elementary concepts as negative numbers. Leibniz wrote "He who understands Archimedes and Apollonius will admire less the achievements of the foremost men of later times."

Chang Tshang (ca 200-142 BC) China

Chinese mathematicians excelled for thousands of years, and were first to discover various algebraic and geometric principles, but they are denied credit because of Western ascendancy. Although there were great Chinese mathematicians a thousand years before the Han Dynasty, and innovations continued for centuries after Han, the textbook *Nine Chapters on the Mathematical Art* has special importance. *Nine Chapters* (known in Chinese as *Jiu Zhang Suan Shu* or *Chiu Chang Suan Shu*) was apparently written during the early Han Dynasty (about 165 BC) by Chang Tshang (also spelled Zhang Cang).

Many of the mathematical concepts of the early Greeks were discovered independently in early China. Chang's book gives methods of arithmetic (including cube roots) and algebra, uses the decimal system with zero and negative numbers, proves the Pythagorean Theorem, and includes a clever geometric proof that the perimeter of a right triangle times the radius of its inscribing circle equals the area of its circumscribing rectangle. (Some of this may have been added after the time of Chang; some additions attributed to Liu Hui are mentioned in his mini-bio.)

Nine Chapters was probably based on earlier books, lost during the great book burning of 212 BC, so Chang himself may not have been the major creative genius. Moreover, important revisions and commentaries were added after Chang, notably by Liu Hui (ca 220-280), so Chang may not be appropriate for a Top Mathematicians List (though he was probably not a mere bureaucrat or copyist, as Liu Hui mentions Chang's skill). Nevertheless his book had immense historical importance: It was the dominant Chinese mathematical text for centuries, and had great influence throughout the Far East. After Chang, Chinese mathematics continued to flourish, discovering trigonometry, matrix methods, the binomial theorem, etc. Some of the teachings made their way to India, and from there to the Islamic world and Europe. There is some evidence that the Hindus borrowed the decimal system itself from books like *Nine Chapters*.

No one person can be credited with the invention of the decimal system, but key roles were played by early Chinese (Chang Tshang and Liu Hui), Brahmagupta (and earlier Hindus including

Aryabhatta), and Leonardo Fibonacci. (After Fibonacci, Europe still did not embrace the decimal system until the works of Vieta, Stevin, and Napier.)

Hipparchus of Nicaea (ca 190-120 BC) Greece/Asia Minor

Ptolemy may be the most famous astronomer before Copernicus, but he borrowed heavily from Hipparchus, who might be considered the greatest astronomer ever. (Late Vedic astronomers, including the 6th-century genius Aryabhatta, borrow much from Ptolemy and Hipparchus.) As a mathematician, Hipparchus developed spherical trigonometry, produced trig tables, and fourteen texts of physics and mathematics nearly all of which have been lost, but which seem to have had great teachings, including much of Newton's Laws of Motion. He invented the circle-conformal stereographic map projection which carries his name. As an astronomer, Hipparchus is credited with the discovery of equinox precession, length of the year, thorough star catalogs, and invention of the armillary sphere and perhaps the astrolabe. He had great historical influence in Europe, India and Persia, at least if credited also with Ptolemy's influence. (Hipparchus himself was influenced by Chaldean astronomers.) Hipparchus' work implies a better approximation to **π** than that of Apollonius, perhaps it was **π ≈ 377/120** as Ptolemy used.

Liu Hui (ca 220-280) China

Liu Hui made major improvements to Chang's influential textbook *Nine Chapters*, making him among the most important of Chinese mathematicians ever. (He seems to have been a much better mathematician than Chang, but just as Newton might have gotten nowhere without Kepler, Vieta, Huygens, Fermat, Wallis, Cavalieri, etc., so Liu Hui might have achieved little had Chang not preserved the ancient Chinese learnings.) Among Liu's achievements are an emphasis on generalizations and proofs, an early recognition of the notions of infinitesimals and limits, the Gaussian elimination method of solving simultaneous linear equations, calculations of solid volumes (including the use of Cavalieri's Principle), and a new method to calculate square roots. Like Archimedes, Liu discovered the formula for a circle's area; however he failed to calculate a sphere's volume, writing "Let us leave this problem to whoever can tell the truth."

Although it was almost child's-play for any of them, Archimedes, Apollonius, and Hipparchus had all improved precision of **π**'s estimate. It seems fitting that Liu Hui did join that select company of record setters: He developed a recurrence formula for regular polygons allowing arbitrarily-close approximations for **π**. He also devised an interpolation formula to simplify that calculation; this yielded the "good-enough" value 3.1416, which is still taught today in primary schools. (Liu's successors applied his method to produce even better approximations, so that country kept the pi-accuracy record for 1100 years.)

Pappus of Alexandria (ca 300) Egypt, Greece

Pappus may have been the greatest Western mathematician during the 14 centuries that separated Apollonius and Fibonacci. He wrote about arithmetic methods, plane and solid geometry, the axiomatic method, celestial motions and mechanics. In addition to his own original research, his texts are noteworthy for preserving works of earlier mathematicians that would otherwise have been lost.

Pappus presents several ingenious geometric theorems including Desargues' Homology Theorem (which Pappus attributes to Euclid), a special case of Pascal's Hexagram Theorem, and Pappus' Theorem itself (two projective pencils can always be brought into a perspective position). For these theorems, Pappus is sometimes called the "Father of Projective Geometry." Other ingenious theorems include an angle trisection method using a fixed hyperbola. He stated (but didn't prove) the *Isoperimetric Theorem*, also writing "Bees know this fact which is useful to them, that the hexagon ... will hold more honey for the same material than [a square or triangle]."

For preserving the teachings of Euclid and Apollonius, as well as his own theorems of geometry, Pappus certainly belongs on a list of great ancient mathematicians. But these teachings lay dormant during Europe's Dark Ages, diminishing Pappus' historical significance.

Mathematicians after Classical Greece

Greece was eventually absorbed into the Roman Empire (with Archimedes himself famously killed by a Roman soldier); Rome did not pursue pure science as Greece had, and eventually Europe fell into a Dark Age. The Greek emphasis on pure mathematics and proofs was key to the future of mathematics, but they were missing an even more important catalyst: a decimal place-value system based on zero and nine other symbols. (It's still hard to believe that this "obvious" and so-convenient system didn't catch on in Europe until almost the Renaissance.)

Aryabhatta (476-550) Ashmaka & Kusumapura (India)

Indian mathematicians excelled for thousands of years, and eventually even developed advanced techniques like Taylor series before Europeans did, but they are denied credit because of Western ascendancy. Among the Hindu mathematicians, Aryabhatta (called Arjehir by Arabs) may be most famous.

While Europe was in its early "Dark Age," Aryabhatta advanced arithmetic, algebra, elementary analysis, and especially trigonometry, using the decimal system. Aryabhatta is sometimes called the "Father of Algebra" instead of al-Khowarizmi (who himself cites the work of Aryabhatta). His most famous accomplishment was the *Aryabhatta Algorithm* (connected to continued fractions) for solving Diophantine equations. Because Aryabhatta built on earlier Indian works (which in turn borrowed from non-Indian sources), and because his works are reported second-hand and often misconstrued, his personal importance may be somewhat exaggerated. It is claimed that Aryabhatta concluded that the planets rotate around the Sun in elliptical orbits, but this is doubtful. He is said to have introduced the constant *e*. He used $\pi \approx 3.1416$; it is unclear whether he discovered this independently or borrowed it from Liu Hui of China.

Brahmagupta `Bhillamalacarya' (589-668) Rajasthan (India)

No one person gets unique credit for the invention of the decimal system but Brahmagupta's textbook *Brahmasphutasiddhanta* was very influential, and is sometimes considered the first textbook "to treat zero as a number in its own right." It also treated negative numbers. (Others claim these were first seen 800 years earlier in Chang Tshang's Chinese text and were implicit in what survives of earlier Hindu works, but Brahmagupta's text discussed them lucidly.)

Brahmagupta Bhillamalacarya (`The Teacher from Bhillamala') made great advances in arithmetic, algebra, numeric analysis, and geometry. Several theorems bear his name, including the formula for the area of a cyclic quadrilateral:

$$16 A^2 = (a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d)$$

Another famous Brahmagupta theorem dealing with such quadrilaterals can be phrased "In a circle, if the chords AB and CD are perpendicular and intersect at E, then the line from E which bisects AC will be perpendicular to BD." Proving Brahmagupta's theorems are good challenges even today.

In addition to his famous writings on practical mathematics and his ingenious theorems of geometry, Brahmagupta solved the general quadratic equation, and worked on Diophantine and Pell's equations. He proved Brahmagupta's identity (the set of sums of two squares is closed under multiplication). He applied mathematics to astronomy, predicting eclipses, etc.

Muhammed `Abu Jafar' ibn Musâ **al-Khowârizmi** (ca 780-850) Persia, Iraq

Al-Khowarizmi was a Persian who worked as a mathematician, astronomer and geographer early in the Golden Age of Islamic science. He introduced the Hindu decimal system to the Islamic world and Europe. He wrote the book *Al-Jabr*, which demonstrated simple algebra and geometry, and several other influential books including ones on calculating with the decimal system, trigonometry, geography, astronomy, the Hebrew calendar, etc. The word *algorithm* is borrowed from Al-Khowarizmi's name. There were several Muslim mathematicians who contributed to the development of Islamic science, and indirectly to Europe's later Renaissance, but Al-Khowarizmi was one of the earliest and most influential.

Alhazen **ibn al-Haytham** (965-1039) Iraq, Egypt

Alhazen ibn al-Haytham (aka Abu Ali Al-asan ibn Alhaitham) made contributions to math, optics, and astronomy which eventually influenced Copernicus, Regiomantus, Kepler and Wallis, among others, thus affecting Europe's Scientific Revolution. He's been called the best scientist of the Middle Ages; his *Book of Optics* has been called the most important physics text prior to Newton; his writings in physics anticipate the Principle of Least Action, Newton's First Law of Motion, and the notion that white light is composed of the color spectrum. His other achievements in optics include improved lens design, an analysis of the camera obscura, an early explanation for the rainbow, and experiments on visual perception. He also did work in human anatomy and medicine. (In a famous leap of over-confidence he claimed he could control the Nile River; when the Caliph ordered him to do so, he then had to feign madness!) Alhazen has been called the "Father of Modern Optics" and, because he emphasized hypotheses and experiments, "The First Scientist."

In number theory, Alhazen worked with perfect numbers, Mersenne primes; and stated Wilson's Theorem (eventually proven by Lagrange). He essentially proved the Power Series Theorem (later attributed to Bernoulli). He solved Alhazen's Billiard Problem (originally posed as a problem in mirror design), a difficult construction which continued to intrigue several great mathematicians including Huygens. Alhazen's attempts to prove the Parallel Postulate make him the earliest important mathematician to investigate non-Euclidean geometry.

Omar al-Khayyám (1048-1123) Persia

Omar Khayyam (aka Ghiyas od-Din Abol-Fath Omar ibn Ebrahim Khayyam Neyshaburi) did clever work with geometry, developing an alternate to Euclid's Parallel Postulate and then deriving the parallel result using theorems based on the *Khayyam-Saccheri quadrilateral*. He derived solutions to cubic equations using the intersection of conic sections with circles. Remarkably, he stated that the cubic solution could not be achieved with straightedge and compass, a fact that wouldn't be proved until the 19th century. Khayyam did even more important work in algebra, writing an influential textbook, and developing new solutions for various higher-degree equations. He discovered the binomial coefficients. His symbol ('shay') for an unknown in an algebraic equation was transliterated to become our 'x'.

Khayyam was also an important astronomer, measuring the year far more accurately than ever before, improving the Persian calendar, and building a famous star map. He emphasized science over religion and proved that the Earth rotates around the Sun. He also wrote treatises on philosophy, music, mechanics and natural science. Despite his great achievements in algebra, geometry, and astronomy, today Omar al-Khayyam is most famous for his rich poetry (*The Rubaiyat of Omar Khayyam*).

Bháscara Áchárya (1114-1185) India

Bhascara (also called Bhaskara II or Bhaskaracharya) may have been the greatest of the Hindu mathematicians. He made achievements in several fields of mathematics including some Europe wouldn't learn until the time of Euler. His textbooks dealt with many matters, including solid geometry, combinations, and advanced arithmetic methods. He was also an astronomer. (It is sometimes claimed that his equations for planetary motions anticipated the Laws of Motion discovered by Kepler and Newton, but this claim is doubtful.) In algebra, he solved various equations including 2nd-order Diophantine, quartic, Brouncker's and Pell's equations. His "Chakravala method," an early application of mathematical induction to solve 2nd-order equations, has been called "the finest thing achieved in the theory of numbers before Lagrange." (Earlier Hindus, including Brahmagupta, contributed to this method.) In several ways he anticipated calculus: he used Rolle's Theorem; he may have been first to use the fact that $d\sin x = \cos x \cdot dx$; and he once wrote that multiplication by **0/0** could be "useful in astronomy." In trigonometry, which he valued for its own beauty as well as practical applications, he developed spherical trig and was first to present the identity

$$\sin a+b = \sin a \cdot \cos b + \sin b \cdot \cos a$$

Bhascara's achievements came centuries before similar discoveries in Europe. It is an open riddle of history whether any of Bhascara's teachings trickled into Europe in time to influence its Scientific Renaissance.

Leonardo `Bigollo' Pisano (Fibonacci) (ca 1170-1245) Italy

Leonardo (known today as Fibonacci) introduced new methods of arithmetic to Europe, and relayed the mathematics of the Hindus, Persians, and Arabs. Others had translated Islamic mathematics, e.g. the works of al-Khowarizmi, into Latin, but Leonardo was the influential teacher. He re-introduced older Greek ideas like Mersenne numbers and Diophantine equations, and made important contributions in number theory, most famously defining *congruums* and proving theorems about them. His work with *congruums*, which arose from the search for three square numbers in consecutive arithmetic series, has been called the finest in number theory before Fermat, and (although this fact is often overlooked) included a proof of the $n = 4$ case of Fermat's Last Theorem. Leonardo's writings cover a very broad range including new theorems of geometry, methods to construct and convert Egyptian fractions (which were still in wide use), irrational numbers, the Chinese Remainder Theorem, theorems about Pythagorean triplets, and the series 1, 1, 2, 3, 5, 8, 13, ... which is now linked with the name Fibonacci.

Leonardo provided Europe with the decimal system, algebra and the 'lattice' method of multiplication, all far superior to the methods then in use. He introduced notation like **3/5**; his clever extension of this for quantities like *5 yards, 2 feet, and 3 inches* is more efficient than today's notation. It seems hard to believe but before the decimal system, mathematicians had no notation for zero. Referring to this system, Gauss was later to exclaim "To what heights would science now be raised if Archimedes had made that discovery!" In addition to his great historic importance and fame (he was a favorite of Emperor Frederick II), Leonardo `Fibonacci' has been called "the greatest number theorist between Diophantes and Fermat" and "the most *talented* mathematician of the Middle Ages."

Many histories describe him as bringing Islamic mathematics to Europe, but in Fibonacci's own preface to *Liber Abaci*, he specifically credits the Hindus:

> ... as a consequence of marvelous instruction in the art, to the nine digits of the Hindus, the knowledge of the art very much appealed to me before all others, and for it I realized that all its aspects were studied in Egypt, Syria, Greece, Sicily, and Provence, with their varying methods;
> ... But all this even, and the algorism, as well as the art of Pythagoras, I considered as almost a mistake in respect to the method of the Hindus. Therefore, embracing more stringently that method of the Hindus, and taking stricter pains in its study, while adding certain things from my own understanding and inserting also certain things from the niceties of Euclid's geometric art, I have striven to compose this book in its entirety as understandably as I could, ...

Had the Scientific Renaissance begun in the Islamic Empire, someone like al-Khowarizmi would have greater historic significance than Fibonacci, but the Renaissance did happen in Europe. *Liber Abaci*'s summary of the decimal system has been called "the most important sentence ever written." Even granting this to be an exaggeration, there is no doubt that the Scientific Revolution owes a huge debt to Leonardo `Fibonacci' Pisano.

Madhava of Sangamagrama (1340-1425) India

Madhava, also known as Irinjaatappilly Madhavan Namboodiri, is considered the founder of the important Kerala school of mathematics and astronomy. If everything credited to him was his own work, he was a truly great mathematician. His analytic geometry preceded and surpassed Descartes', and included differentiation and integration. He also did work with continued fractions, trigonometry, and geometry.

Madhava is most famous for his work with Taylor series, discovering identities like $\textbf{\textit{sin}}\ \mathbf{q = q - q^3/3! + q^5/5! - ...}$, formulae for π, including the one attributed to Leibniz, and the then-best known approximation $\pi \approx \mathbf{104348 / 33215}$. (From Wikipedia we see this record was subsequently broken by relative unknowns: a Persian in 1424, a German ca. 1600, John Machin 1706. In 1949 the π calculation record was held briefly by John von Neumann and the ENIAC.)

Girolamo **Cardano** (1501-1576) Italy

Girolamo Cardano (or Jerome Cardan) was a highly respected physician and was first to describe typhoid fever. He was also an accomplished gambler and chess player and wrote an early book on probability. He was also a remarkable inventor: the combination lock, an advanced gimbal, a ciphering tool, and the Cardan shaft with universal joints are all his inventions and are in use to this day. (The U-joint is sometimes called the Cardan joint.) He also helped develop the camera obscura, and proved the theorem of geometry underlying the 2:1 spur wheel which converts circular to reciprocal rectilinear motion. Cardano made contributions to physics; he noted that projectile trajectories are parabolas, and may have been first to note the impossibility of perpetual motion machines. He did work in philosophy, geology, hydrodynamics, music; he wrote books on medicine and an encyclopedia of natural science.

But Cardano is most remembered for his achievements in mathematics. He was first to publish general solutions to cubic and quartic equations (though these were largely based on others' work). He introduced complex numbers, although he did not develop their theory further. He introduced binomial coefficients and the binomial theorem, and introduced and solved the geometric hypocyloid problem, as well as other geometric theorems. Da Vinci and Galileo may have been more influential than Cardano, but of the three great medieval generalists who preceded Kepler, it seems clear that Cardano was the most accomplished mathematician.

Cardano's life had tragic elements. Throughout his life he was tormented that his father (a friend of Leonardo da Vinci) married his mother only after Cardano was born. (And his mother tried several times to abort him.) Cardano's reputation for gambling and aggression interfered with his career. He practiced astrology and was imprisoned for heresy when he cast a horoscope for Jesus. His son apparently murdered his own wife. Leibniz wrote of Cardano: "Cardano was a great man with all his faults; without them, he would have been incomparable."

François **Viète** (1540-1603) France

Francois Viete (or Franciscus Vieta) was a French nobleman and lawyer who was a favorite of King Henry IV and eventually became a royal privy councillor. In one notable accomplishment he broke the Spanish diplomatic code, allowing the French government to read Spain's messages and publish a secret Spanish letter; this apparently led to the end of the Huguenot Wars of Religion.

More importantly, Vieta was certainly the best French mathematician prior to Descartes and Fermat. He laid the groundwork for modern mathematics; his works were studied by Isaac Newton. In his role as a young tutor Vieta used decimal numbers before they were popularized by Simon Stevin and may have guessed that planetary orbits were ellipses before Kepler. Vieta did work in geometry, reconstructing and publishing proofs for Apollonius' lost theorems. He discovered several trigonometric identities including the Law of Cosines and a generalization of Ptolemy's Formula, the latter (then called *prosthaphaeresis*) providing a calculation shortcut similar to logarithms in that multiplication is reduced to addition (or exponentiation reduced to multiplication). Such trigonometric formulae revolutionized calculations and may even have helped stimulate the development and use of logarithms by Napier and Kepler. He developed the first infinite-product formula for **π**. Vieta is most famous for his systematic use of decimal notation and variable letters, for which he is sometimes called the Father of Modern Algebra. (Vieta used A,E,I,O,U for unknowns and consonants for parameters; it was Descartes who first used X,Y,Z for unknowns and A,B,C for parameters.) In his works Vieta emphasized the relationships between algebraic expressions and geometric constructions. One key insight he had is that addends must be homogeneous (i.e., "apples shouldn't be added to oranges"), a seemingly trivial idea but which can aid intuition even today.

Descartes, who once wrote "I began where Vieta finished," is now extremely famous, while Vieta is much less known. (He isn't *even mentioned once* in Bell's famous *Men of Mathematics*.) Many would now agree this is due in large measure to Descartes' deliberate deprecations of competitors in his quest for personal glory. (Vieta wasn't particularly humble either, calling himself the "French Apollonius.")

```
    PI := 2
    Y  := 0
  LOOP:
    Y  := SQRT(Y + 2)
    PI := PI * 2 / Y
    IF (more precision needed) GOTO LOOP
```

Vieta's formula for **π** is clumsy even with modern notation. Easiest may be to consider it the result of the BASIC program above.

John **Napier** 8th of Merchistoun (1550-1617) Scotland

Napier was a Scottish Laird who was a noted theologian and thought by many to be a magician (his nickname was Marvellous Merchiston). Today, however, he is best known for his work with *logarithms*, a word he invented. He published the first large table of logarithms and also helped popularize usage of the decimal point and lattice multiplication. He invented *Napier's Bones*, a crude hand calculator which could be used for division and root extraction, as well as multiplication.

Although he was certainly very clever (and had novel mathematical insights not mentioned in this summary), Napier proved no deep theorem and may not belong on a list of great mathematicians. Nevertheless, his revolutionary methods of arithmetic had immense historical importance and led to the Scientific Revolution.

Johannes **Kepler** (1571-1630) Germany

Kepler was interested in astronomy as a very young lad, became a professor of mathematics, then Tycho Brahe's understudy, and, on Brahe's death, was appointed Imperial Mathematician at the age of twenty-nine. His observations of the planets with Brahe, along with his study of Apollonius' 1800-year old work, led to Kepler's three Laws of Planetary Motion, which in turn led directly to Newton's Laws of Motion. As one of the key figures in the Scientific Revolution, he ranks #75 on Michael Hart's famous list of the Most Influential Persons in History. He did other work in physics, especially in optics which he needed to develop for his telescopes.

According to Kepler's Laws, the planets move at variable speed along ellipses. The Earth-bound observer is himself describing such an orbit and in almost the same plane as the planets; thus discovering the Laws would be a difficult challenge even for someone armed with computers and modern mathematics. (The very famous Kepler Equation relating a planet's eccentric and anomaly is just one tool Kepler needed to develop.) Kepler understood the importance of his remarkable discovery, even if contemporaries like Galileo did not, writing:

> "I give myself up to divine ecstasy ... My book is written. It will be read either by my contemporaries or by posterity   I care not which. It may well wait a hundred years for a reader, as God has waited 6,000 years for someone to understand His work."

Besides the trigonometric results needed to discover his Laws, Kepler made other contributions to mathematics. He was first to notice that the set of Platonic regular solids was incomplete if concave solids are admitted. He may have been first to observe that the ratio of Fibonacci numbers converges to the *Golden Mean*. His writings include mensuration methods which preceded and inspired Cavalieri and represent a precursor to calculus; he even anticipated Fermat's formula $df(x)/dx = 0$ for finding function extrema. Kepler reasoned that the structure of snowflakes was evidence for the then-novel atomic theory of matter. He noted that the obvious packing of cannonballs gave maximum density (this became known as "Kepler's Conjecture" and wasn't proved rigorously until the 20th century). In addition to his physics and mathematics, Kepler was an astrologer and mystic; he had ideas similar to Pythagoras about numbers ruling the cosmos (writing that the purpose of studying the world "should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics"). Kepler's mystic beliefs even led to his own mother being imprisoned for witchcraft!

Johannes Kepler (along with Galileo, Fermat and perhaps Huygens, Wallis and Vieta) is among the giants on whose shoulders Newton was proud to stand. Yet, despite his outstanding genius, intuition and perspicacity, Kepler's mathematical accomplishments may not seem enough to merit a place on this List. Several great physicists were greater mathematical geniuses than Kepler, e.g. Heisenberg, Schrodinger, Dirac, Pauli, Witten. Nevertheless I include Kepler because of his immense historical importance. (Some historians place him ahead of Galileo and Copernicus as the single most important early contributor to the Scientific Revolution.) It would be a shame to reject him from this list on a technicality!

René **Déscartes** (1596-1650) France

Descartes' early career was that of soldier-adventurer and he finished as tutor to royalty, but in between he achieved fame as the preeminent intellectual of his day. He invented analytic geometry and is therefore called the "Father of Modern Mathematics." Because of his famous philosophical writings ("Cogito ergo sum") he is considered, along with Aristotle, to be one of the most influential thinkers in history. (He ranks #49 on Michael Hart's famous list of the Most

Influential Persons in History.) Descartes made important contributions to physics (e.g. the law of conservation of momentum), and mathematical notation (e.g. the use of superscripts to denote exponents). His famous mathematical theorems include the Rule of Signs (for determining the signs of polynomial roots), the elegant formula relating the radii of *Soddy kissing circles*, and an improvement on the ancient construction method for cube-doubling.

Descartes has an extremely high reputation and would be ranked higher by most list makers. I've demoted him partly because he had only insulting things to say about Pascal and Fermat, each of whom was more brilliant at mathematics than Descartes. (Some even suspect that Descartes arranged the destruction of Pascal's lost *Essay on Conics*.) Anyway, the historical importance of these Frenchmen may be slightly exaggerated since others, e.g. Cavalieri, were also on the verge of developing modern mathematics.

Francesco Bonaventura de **Cavalieri** (1598-1647) Italy

Cavalieri did work in analysis, geometry and trigonometry; he is most famous for developing a rudimentary calculus. (Because of his calculus, Cavalieri is often listed among greatest mathematicians; however his work was largely anticipated by Kepler, and was soon surpassed by Fermat and Wallis.) Cavalieri also worked in theology, astronomy, mechanics and optics; he was an inventor, and published logarithm tables. He wrote several books, the first one developing the properties of mirrors shaped as conic sections. His name is especially remembered for Cavalieri's Principle of Solid Geometry. Galileo said of Cavalieri, "Few, if any, since Archimedes, have delved as far and as deep into the science of geometry."

Pierre de **Fermat** (1601-1665) France

Pierre de Fermat was the most brilliant mathematician of his era and, along with Descartes, one of the most influential. Although mathematics was just his hobby (Fermat was a government lawyer), Fermat practically founded Number Theory, and also played key roles in the discoveries of Analytic Geometry and Calculus. He was also an excellent geometer (e.g. discovering a triangle's *Fermat point*), and (in collaboration with Blaise Pascal) discovered probability theory. Fellow geniuses are the best judges of genius, and Blaise Pascal had this to say of Fermat: "For my part, I confess that [Fermat's researches about numbers] are far beyond me, and I am competent only to admire them." E.T. Bell wrote "it can be argued that Fermat was at least Newton's equal as a pure mathematician."

Fermat's most famous discoveries in number theory include the ubiquitously-used *Fermat's Little Theorem*, the $n = 4$ and $n = 3$ cases of his conjectured *Fermat's Last Theorem*, and *Fermat's Christmas Theorem* (that any prime (4n+1) can be represented as the sum of two squares in exactly one way, also called the *Fermat-Euler Prime Number Theorem*). As suggested by the "Euler" in the name of this latter theorem (which Fermat records proving with difficulty using "infinite descent"), proofs for this and many other Fermat results had to be rediscovered (most of Fermat's work was never published). However it is wrong to suppose that Fermat's work comprised mostly false or unproven conjectures. (This misconception arises from his so-called "Last Theorem" which was actually just a private scribble.)

Fermat developed a system of analytic geometry which both preceded and surpassed that of Descartes; he developed methods of differential and integral calculus which Newton acknowledged as an inspiration. Solving **$df(x)/dx = 0$** to find extrema of **$f(x)$** is perhaps the most useful idea in applied mathematics; this technique originated with Fermat. Fermat was also the first European to find the integration formula for the general polynomial; he used his calculus to find centers of gravity, etc. Fermat anticipated the principle of least action (which Leibniz, Maupertius, Euler, Lagrange and Hamilton would later develop) and used it to establish basic principles of optics.

Fermat's contemporaneous rival Rene Descartes is more famous than Fermat, and Descartes' writings were more influential. Whatever one thinks of Descartes as a *philosopher*, however, it seems clear that Fermat was the better *mathematician*. Fermat and Descartes independently discovered analytic geometry, but it was Fermat who extended it to more than 2 dimensions, and followed up by developing elementary calculus. Fermat and Descartes did work in physics and independently discovered the (trigonometric) law of refraction, but only Fermat had the insight to realize that the refraction law implied that light has a finite speed !

John Brehaut **Wallis** (1616-1703) England

Wallis began his life as a savant at arithmetic, a medical student (he may have contributed to the concept of blood circulation), and theologian, but went on to become the most brilliant and influential English mathematician before Newton. He made major advances in analytic geometry, but also contributions to algebra, geometry and trigonometry. He is especially famous for introducing fractional and negative exponents, taking the area of curves, and treating inelastic collisions. He was the first European to solve Pell's Equation. Like Vieta, Wallis was a code-breaker, helping the Commonwealth side (though he later petitioned against the beheading of King Charles I). He was first to use the symbol ∞, and coined the term "continued fraction."

Also like Vieta, Wallis created an infinite product formula for pi, which might be (but isn't!) written today as:

$$\pi = 2 \prod_{k=1,\infty} 1+(4k^2-1)^{-1}$$

Blaise **Pascal** (1623-1662) France

Pascal was an outstanding genius who studied geometry as a child. At the age of sixteen he stated and proved Pascal's Theorem, a fact relating any six points on any conic section. The Theorem is sometimes called the "Cat's Cradle" or the "Mystic Hexagram." Pascal followed up this result by showing that each of Apollonius' famous theorems about conic sections was a corollary of the Mystic Hexagram; along with Gerard Desargues (1591-1661), he was a key pioneer of projective geometry. Returning to geometry late in life, Pascal advanced the theory of the cycloid. In addition to his work in classic and projective geometry, he founded probability theory, made contributions to axiomatic theory, and the invention of calculus. His name is associated with the Pascal's Triangle of combinatorics and Pascal's Wager in theology.

Like most of the greatest mathematicians, Pascal was interested in physics and mechanics, studying fluids, explaining vacuum, and inventing the syringe and hydraulic press. At the age of eighteen he designed and built the world's first automatic adding machine. (Although he continued to refine this invention, it was never a commercial success. Pascal abandoned mathematics for religion, suffered poor health, and died at an early age.)

Christiaan **Huygens** (1629-1695) Holland, France

Christiaan Huygens (or Hugens or Huyghens) was one of the greatest scientists during Europe's Renaissance. He was an excellent mathematician, but is more famous for his physical theories and inventions. He developed laws of motion before Newton, including the inverse-square law of gravitation, centripetal force, and treatment of solid bodies rather than point approximations; he corrected a deficiency in Descartes' law of momentum conservation. He advanced the wave ("undulatory") theory of light, a key concept being *Huygen's Principle*, that each point on a wave front acts as a new source of radiation. His optical discoveries include explanations for polarization and phenomena like haloes. (Because of Newton's high reputation and corpuscular theory of light, Huygens' superior wave theory was largely ignored until the 19th-century work of Young and Fresnel. Later, Einstein, partly anticipated by Hamilton, developed the modern notion of wave-particle duality.)

Huygens is famous for his inventions of clocks and lenses. He designed the first reliable pendulum clock, and the first balance spring watch, which he presented to his patron, King Louis XIV of France. He invented superior lens grinding techniques, the achromatic eye-piece, and the best telescope of his day. He was himself a famous astronomer: he discovered Titan and was first to properly describe Saturn's rings and the Orion Nebula. He also designed, but never built, an internal combustion engine. He promoted the use of 31-tone music: a 31-tone organ was in use in Holland as late as the 20th century. Huygens was an excellent card player, billiard player, horse rider, and wrote a book speculating about extra-terrestrial life.

As a mathematician, Huygens did brilliant work in geometry and analysis, proving theorems about conic sections, the cycloid and the catenary. He was first to show that the cycloid solves the tautochrone problem; he used this fact to design an (impractical) compound pendulum clock that would be more accurate than an ordinary pendulum clock. He was first to find the flaw in Saint-Vincent's then-famous circle-squaring method; Huygens himself solved some related quadrature problems. He introduced the concepts of evolute and involute. His friendships with Descartes, Pascal, Mersenne and others helped inspire his mathematics; Huygens in turn was inspirational to the next generation. At Pascal's urging, Huygens published the first real textbook on probability theory; he also became the first practicing actuary.

Huygens had tremendous creativity, historical importance, and depth and breadth of genius, making him a prime candidate for this List despite that his emphasis was physics rather than pure mathematics. But one of his most important feats was serving as tutor to the otherwise self-taught Gottfried Leibniz (who'd "wasted his youth" without learning any math). Before agreeing to tutor him, Huygens tested the 25-year old Leibniz by asking him to sum the reciprocals of the triangle numbers.

Takakazu **Seki** (Kowa) (1637?-1708) Japan

Seki Takakazu was a self-taught prodigy who developed a new notation for algebra, and made several discoveries before Western mathematicians did; these include determinants, the Newton-Raphson method, Newton's interpolation formula, Bernoulli numbers, discriminants, methods of calculus, and probably much that has been forgotten (Japanese schools practiced secrecy). He calculated π to ten decimal places using Aitkin's method (rediscovered in the 20th century). He also worked with magic squares. He is remembered as a brilliant genius and very influential teacher.

Seki's work was not propagated to Europe, so has minimal historic importance; otherwise Seki might rank high on our list.

Isaac (Sir) **Newton** (1642-1727) England

Newton was an industrious lad who built marvelous toys (e.g. a model windmill powered by a mouse on treadmill). His genius seems to have blossomed at about age 22 when, on leave from University, he began revolutionary advances in mathematics, optics, dynamics, thermodynamics, acoustics and celestial mechanics. He is most famous for his Three Laws of Motion (inertia, force, reciprocal action) and Law of Universal Gravitation. As Newton himself acknowledged, the Laws weren't fully novel: Hipparchus, Ibn al-Haytham, Galileo and Huygens had all developed much of it already, and Newton credits the First Law itself to Aristotle. (Newton's other intellectual interests included chemistry, theology, astrology and alchemy.) Although this list is concerned only with mathematics, Newton's greatness is indicated by the wide range of his physics: even without his revolutionary Laws of Motion and his Cooling Law of thermodynamics, he'd be famous just for his work in optics, where he explained diffraction and observed that white light is a mixture of all the rainbow's colors. (Although his corpuscular theory competed with Huygen's wave theory, Newton understand that his theory was incomplete without waves, and thus anticipated wave-particle duality.) Newton also designed the first reflecting telescope, first reflecting microscope, and the sextant.

Although others also developed the techniques independently, Newton is regarded as the Father of Calculus (which he called "fluxions"); he shares credit with Leibniz for the Fundamental Theorem of Calculus (that integration and differentiation are each other's inverse operation). He applied calculus for several purposes: finding areas, tangents, the lengths of curves and the maxima and minima of functions. In addition to several other important advances in analytic geometry, his mathematical works include the Binomial Theorem, his eponymous numeric method, the idea of polar coordinates, and power series for exponential and trigonometric functions. (His equation $e^x = \sum x^k / k!$ has been called the "most important series in mathematics.") He contributed to algebra and the theory of equations, proving facts about cubic equations, attempting generalization of Descartes' rule of signs, etc. He proved that same-mass spheres of any radius have equal gravitational attraction, a key to celestial motions. (Like some of the greatest ancient mathematicians, Newton took the time to compute an approximation to π. His was perhaps the best-yet by a European, but not as accurate as al-Kashi's.)

Newton is so famous for his calculus, optics and laws of motion, it is easy to overlook that he was also one of the greatest geometers. Among many marvelous theorems, he proved several about quadrilaterals and their in- or circum-scribing ellipses, and constructed the parabola defined by four given points. An anecdote often cited to demonstrate his brilliance is the problem of the *brachistochrone*, which had baffled the best mathematicians in Europe, and came to Newton's attention late in life. He solved it in a few hours and published the answer anonymously. But on seeing the solution Jacob Bernoulli immediately exclaimed "I recognize the lion by his footprint."

In 1687 Newton published *Philosophiae Naturalis Principia Mathematica*, surely the greatest scientific book ever written. The motion of the planets was not understood before Newton, although the *heliocentric* system allowed Kepler to describe the orbits. In *Principia* Newton analyzed the consequences of his Laws of Motion and introduced the Law of Universal Gravitation. The notion that the Earth rotated about the Sun was introduced in ancient Greece, but Newton explained *why* it did, and the Great Scientific Revolution began. Newton once wrote "Truth is ever to be found in the simplicity, and not in the multiplicity and confusion of things." Sir Isaac Newton was buried at Westminster Abbey in a tomb inscribed "Let mortals rejoice that so great an ornament to the human race has existed."

Newton ranks #2 on Michael Hart's famous list of the Most Influential Persons in History. (Muhammed the Prophet of Allah is #1.) Whatever the criteria, Newton would certainly rank first, or behind only Einstein, on any list of physicists, or scientists in general, but some listmakers would demote him slightly on a list of pure mathematicians: his emphasis was physics not mathematics, and the contribution of Leibniz (Newton's rival for the title *Inventor of Calculus*) lessens the historical importance of Newton's calculus. One reason I've ranked him at #1 is a comment by Gottfried Leibniz himself: "Taking mathematics from the beginning of the world to the time when Newton lived, what he has done is much the better part."

Gottfried Wilhelm von **Leibniz** (1646-1716) Germany

Leibniz was one of the most brilliant and prolific intellectuals ever; and his influence in mathematics (especially his co-invention of the infinitesimal calculus) was immense. His childhood IQ has been estimated as second-highest in all of history, behind only Goethe. Descriptions which have been applied to Leibniz include "one of the two greatest universal geniuses" (da Vinci was the other); "the most important logician between Aristotle and Boole;" and the "Father of Applied Science."

Mathematics was just a self-taught sideline for Leibniz, who was a philosopher, lawyer, historian, diplomat and renowned inventor. Because he "wasted his youth" before learning mathematics, he probably ranked behind the Bernoullis as well as Newton in pure mathematical talent, and thus he may be the only mathematician among the Top Twenty who was never the greatest living algorist or theorem prover. We won't try to summarize Leibniz' contributions to philosophy and diverse other fields including biology; as just three examples: he predicted the Earth's molten core, introduced the notion of subconscious mind, and built the first calculator that could do multiplication.

Leibniz pioneered the common discourse of mathematics, including its continuous, discrete, and symbolic aspects. (His ideas on symbolic logic weren't pursued and it was left to Boole to reinvent this almost two centuries later.) Mathematical innovations attributed to Leibniz include the symbols $\int$, $df(x)/dx$; the terms "function" and "analysis situ;" the concepts of matrix determinant and Gaussian elimination; the theory of geometric envelopes; and the binary number system. His works seem to anticipate cybernetics and information theory; and Mandelbrot acknowledged Leibniz' anticipation of self-similarity. Like Newton, Leibniz discovered The Fundamental Theorem of Calculus; his contribution to calculus was much more influential than Newton's, and his superior notation is used to this day. As Leibniz himself pointed out, since the concept of mathematical analysis was already known to ancient Greeks, the revolutionary invention was *notation* ("calculus"), because with "symbols [which] express the exact nature of a thing briefly ... the labor of thought is wonderfully diminished."

Leibniz' thoughts on mathematical physics had some influence. He developed laws of motion that gave different insights from those of Newton. Although his argumentation was partly based on theology, he anticipated the Principle of Least Action that would be developed by Maupertuis, Lagrange and Hamilton. His cosmology was opposed to that of Newton but, anticipating theories of Mach and Einstein, is more in accord with modern physics. Mathematical physicists influenced by Leibniz include not only Mach, but perhaps Hamilton and Poincare themselves.

Although others found it independently (including perhaps Madhava three centuries earlier), Leibniz discovered and proved a striking identity for **π**:

π/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - ...

Jacob & Johann **Bernoulli** (1654-1705; 1667-1748) Switzerland

Jacob and Johann Bernoulli were probably the two greatest mathematicians during the generations between Newton/Leibniz and Leonhard Euler. They were key pioneers in several developments of calculus: Jacob develop important methods for integral and differential equations, coining the word "integral;" Johann advanced exponential calculus; together they founded the calculus of variations. They did significant work in other fields of math and physics as well. Jacob studied Wallis and Barrow, and became friends with Leibniz. Johann learned from his older brother and Leibniz, and went on to become principle teacher to Leonhard Euler. Johann's son Daniel (1700-1782), "the Founder of Mathematical Physics" and discoverer of the Bernoulli Principle underlying airflight, was closer to Euler in age and his close friend. (Several other Bernoullis were outstanding mathematicians, e.g. Nicholaus Bernoulli born 1687 who held Galileo's chair, proved the Bernoulli-Euler Misaddressed Envelope Theorem, posed the famous St. Petersburg Paradox, and still may rank only about 6th-greatest among Bernoullis.)

Jacob liked to pose and solve physical optimization problems. His "catenary" problem (what shape does a clothesline take?) became more famous than the "tautochrone" solved by Huygens. Perhaps the most famous of such problems was the brachistochrone, wherein Jacob recognized Newton's "lion's paw", and about which Johann wrote: "You will be petrified with astonishment [that] this same cycloid, the tautochrone of Huygens, is the brachistochrone we are seeking." Johann solved the catenary before Jacob did; this led to a famous rivalry in the Bernoulli family. (No joint papers were written; instead the Bernoullis, especially Johann, began claiming each others' work.)

Jacob did significant work outside calculus; in fact his most famous work was the *Art of Conjecture*, a textbook on probability and combinatorics which proves the Law of Large Numbers, the Power Series Equation, and introduces the Bernoulli numbers. Jacob also did outstanding work in geometry, for example constructing perpendicular lines which quadrisect a triangle. Although Jacob may have demonstrated greater breadth, his younger brother had no less skill than Jacob, contributed more to calculus, discovered L'Hopital's Rule before L'Hopital did, and made important contributions in physics, e.g. about vibrations, elastic bodies, optics, tides, and ship sails.

It may not be clear which Bernoulli was the "greatest." Johann has special importance as tutor to Leonhard Euler, but Jacob has special importance as tutor to his brother Johann!

Leonhard **Euler** (1707-1783) Switzerland

Euler made decisive contributions in all areas of mathematics; he gave the world modern trigonometry. Just as Archimedes extended Euclid's geometry to marvelous heights, so Euler took marvelous advantage of the analysis of Newton and Leibniz. He was the most prolific mathematician in history and the best algorist. His colleagues called him "Analysis Incarnate." (Laplace, famous for denying credit to fellow mathematicians, once said "Read Euler: he is our master in everything.") Although he emphasized pure mathematics, Euler made several important advances in physics, e.g. extending Newton's Laws of Motion to rotating rigid bodies.

Euler was supreme at discrete mathematics, as well as continuous: He invented graph theory and generating functions. Two of the most important advances in 18th century were Lagrange's calculus of variations and Fourier's spectral series: in each case the key initial discovery was actually Euler's. Euler was also first to prove several interesting theorems of geometry, including facts about the *9-point Feuerbach circle*; relationships among a triangle's altitudes, medians, and circumscribing and inscribing circles; and an expression for a tetrahedron's area in terms of its sides. He was a major figure in number theory, proving the divergence of the sum of prime reciprocals, finding both the largest then-known prime and the largest then-known perfect number, proving **e** to be irrational, proving that all even perfect numbers must have the Mersenne number form that Euclid had discovered 2000 years earlier, and much more. On a lighter note, he constructed a particularly "magical" magic square. Euler ranks #77 on Michael Hart's famous list of the Most Influential Persons in History.

Four of the most important constant symbols in mathematics (π, e, $i = \sqrt{-1}$, and $\gamma = 0.57721566...$) were all introduced or popularized by Euler. He is particularly famous for unifying the trigonometric and exponential functions with the equation: $e^{ix} = \cos x + i \sin x$.

Euler combined his brilliance with phenomenal concentration. He developed the first method to estimate the Moon's orbit (the three-body problem which had stumped Newton), and he settled an arithmetic dispute involving 50 decimal places of a long convergent series. Both these feats were accomplished when he was totally blind. (About this he said "Now I will have less distraction.")

Among many famous and important identities which Euler was first to state and prove is $\zeta(s) = \prod(1-p^{-s})^{-1}$, where the right-side product is taken over all primes p, and where $\zeta(s) = \sum k^{-s}$ is what eventually came to be called *Riemann's zeta function*.

As a young student of the Bernoulli family, Euler discovered and proved the following:

$$\pi^2/6 = \zeta(2) = 1^{-2} + 2^{-2} + 3^{-2} + 4^{-2} + ...$$

This striking identity catapulted Euler to instant fame, since the right-side infinite sum was a famous problem of the time.

Some of Euler's greatest formulae can be combined into curious-looking formulae for π: $\pi^2 = -\log^2(-1) = 6 \prod_{p \in Prime}(1-p^{-2})^{-1/2}$

Joseph-Louis (Comte de) **Lagrange** (1736-1813) Italy, France

Joseph-Louis Lagrange (born Giuseppe Lodovico Lagrangia) was a brilliant man who advanced to become a teen-age Professor shortly after first studying mathematics. He excelled in all fields of analysis and number theory; he made key contributions to the theories of determinants, continued fractions, and many other fields. He invented partial differential equations, and the calculus of variations. He proved a fundamental Theorem of Group Theory, as well as two number theory theorems of great historic interest: Wilson's prime-number theorem, and the fact that every positive integer is the sum of four squares. He laid the foundations for the theory of polynomial equations which Cauchy, Abel, Galois and Poincaré would later complete.

Lagrange's many contributions to physics include understanding of vibrations (he found an error in Newton's work and published the definitive treatise on sound), celestial mechanics (including an explanation of why the Moon keeps the same face pointed towards the Earth), the *principle of least action* (which Hamilton compared to poetry), and the discovery of the Lagrangian points (e.g., in Jupiter's orbit). Lagrange's textbooks were noted for clarity and inspired most of the 19th century mathematicians on this list. Unlike Newton, who used calculus to derive his results but then worked backwards to create geometric proofs for publication, Lagrange relied only on analysis. "No diagrams will be found in this work" he wrote in the preface to his masterpiece *Mécanique analytique*.

Lagrange once wrote "As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection." Both W.W.R. Ball and E.T. Bell, renowned mathematical historians, bypass Euler to name Lagrange as "the Greatest Mathematician of the 18th Century."

Gaspard **Monge** (1746-1818) France

Gaspard Monge, son of a humble peddler, was an industrious and creative inventor who astounded early with his genius, becoming a professor of physics at age 16. As a military engineer he developed the new field of descriptive geometry, so useful to engineering that it was kept a military secret for 15 years. Monge eventually became a close friend and companion of Napoleon Bonaparte, and demonstrated great courage on several occasions. Monge is most famous for laying the foundation for differential geometry. He also did work in discrete math, and differential equations, and anticipated Poncelet's Principle of Continuity. Monge's most famous theorems of geometry are the "Three Circles Theorem" and "Four Spheres Theorem." While his work in descriptive geometry has little interest to pure mathematics, his application of calculus to the curvature of surfaces inspired Gauss and eventually Riemann.

Monge was an inspirational teacher whose students included Fourier, Sophie Germain, Chasles, Brianchon, Ampere, Carnot, Poncelet and several other famous mathematicians. Chasles reports that Monge never drew figures in his lectures, but could make "the most complicated forms appear in space ... with no other aid than his hands, whose movements admirably supplemented his words." The contributions of Poncelet may be more important than those of Monge, but Monge demonstrated great genius as an untutored child, while Poncelet's skills probably developed due to his great teacher. The great Lagrange said "With [Monge's] application of analysis to geometry this devil of a man will make himself immortal."

Pierre-Simon (Marquis de) **Laplace** (1749-1827) France

Laplace was the preeminent mathematical astronomer, and is often called the "French Newton." His masterpiece was *Mecanique Celeste* which redeveloped and improved Newton's work on planetary motions using calculus. While Newton had shown that the two-body gravitation problem led to orbits which were ellipses (or other conic sections), Laplace was more interested in the much more difficult problems involving three or more bodies. (Would Jupiter's pull on Saturn eventually propel Saturn into a closer orbit, or was Saturn's orbit stable for eternity?) Laplace's equations had the optimistic outcome that the solar system was stable.

Laplace advanced the nebular hypothesis of solar system origin, and was first to conceive of black holes and multiple galaxies. He explained the so-called secular acceleration of the Moon. (Today we know Laplace's theories do not fully explain the Moon's path, nor guarantee orbit stability.) His other accomplishments in physics include theories about the speed of sound and surface tension. He was noted for his strong belief in determinism, famously replying to Napoleon's question about God with: "I have no need of that hypothesis."

Laplace viewed mathematics as just a tool for developing his physical theories. Nevertheless, he made many important mathematical discoveries and inventions, most notably the Laplace Transform. He was the premier expert at differential and difference equations, and definite integrals. He developed spherical harmonics, potential theory, the theory of determinants, and advanced Euler's technique of generating functions. In the fields of probability and statistics he made important advances: he proved the Law of Least Squares, and introduced the controversial ("Bayesian") rule of succession. In the theory of equations, he was first to prove that any polynomial of even degree must have a real quadratic factor.

Others might place Laplace higher on the List, but he proved no fundamental theorems of *pure* mathematics (though his partial differential equation for fluid dynamics is one of the most famous in physics), founded no major branch of pure mathematics, and wasn't particularly concerned with rigorous proof. (He is famous for skipping difficult proof steps with the phrase "It is easy to see".) Nevertheless he was surely one of the greatest *applied* mathematicians ever.

Adrien Marie **Legendre** (1752-1833) France

Legendre was an outstanding mathematician who might have been 2nd best in the world very briefly, during Gauss' childhood, but he was not quite in the "same league" as Gauss and Lagrange. Legendre did do important work in plane and solid geometry, spherical trigonometry, number theory, celestial mechanics and other areas of physics. He also made important contributions in several areas of analysis; the notation for partial derivatives is due to him. He invented the Legendre symbol; invented the study of zonal harmonics; proved that π and π^2 were irrational (the former had already been proved by Lambert); and wrote important textbooks in several fields. Although he never accepted non-Euclidean geometry, and had spent much time trying to prove the Parallel Postulate, his inspiring geometry text remained a standard until the 20th century. As one of France's premier mathematicians, Legendre did other important work, promoting the career of Lagrange, developing trig tables, geodesic projects, etc.

There are several important Theorems proposed by Legendre for which he is denied credit, either because his proof was incomplete or was preceded by another's. He proposed and almost proved the Law of Quadratic Reciprocity before Gauss did; proposed the famous theorem about primes in a progression which was proved by Dirichlet; proved and used the important Law of Least Squares which Gauss had left unpublished; proved the N=5 case of Fermat's Last Theorem which is credited to Dirichlet; proposed the famous Prime Number Theorem which was finally proved by Hadamard; and developed various techniques commonly credited to Laplace. (Legendre also conjectured that there is a prime between any $\mathbf{n^2}$ and $\mathbf{(n+1)^2}$: this remains unproven.) Legendre's work in the theory of equations and elliptic integrals directly inspired the achievements of Galois and Abel (which then obsoleted much of Legendre's own work); Chebyshev's work also built on Legendre's foundations. (Given the "also-ran" character of Legendre's career, it is ironic that he ends up as an "also-ran" on this Top Sixty list.)

Jean Baptiste Joseph **Fourier** (1768-1830) France

Joseph Fourier had a varied career: precocious orphan, theology student, young professor of mathematics (advancing the theory of equations), then revolutionary activist. Under Napoleon he was a brilliant and important teacher and historian; accompanied the French Emperor to Egypt; and did excellent service as district governor of Grenoble. In his spare time at Grenoble he continued the work in mathematics and physics that led to his immortality. After the fall of Napoleon, Fourier exiled himself to England, but returned to France when offered an important academic position and published his revolutionary treatise on the Theory of Heat. He is also noted for the notion of dimensional analysis, was first to describe the Greenhouse Effect, and continued his earlier brilliant work with equations.

Fourier's claim to greatness rests on one particular technique: his use of trigonometric series (now called *Fourier series*) in the solution of differential equations. Since "Fourier" analysis is in extremely common use among applied mathematicians, he joins the select company of the eponyms of "Cartesian" coordinates, "Gaussian" curve, and "Boolean" algebra. Because of the importance of Fourier analysis, many listmakers would rank Fourier much higher than I have done; however the work was not exceptional as *pure* mathematics. Fourier's Heat Equation built on Newton's Law of Cooling; and the Fourier series solution itself had already been introduced by Euler and Lagrange. (Another mathematical physicist with greatness due to intuition rather than rigor was Oliver Heaviside (1850-1925).)

Fourier's solution to the heat equation was counterintuitive (heat transfer doesn't seem to involve the oscillations fundamental to trignometric functions): The brilliance of Fourier's imagination is indicated in that the solution had been *rejected* by Lagrange himself. Although a rigorous Fourier Theorem was left for Dirichlet, it has been said that it was Fourier's "very disregard for rigor" that led to his great achievement, which Lord Kelvin compared to poetry.

Johann Carl Friedrich **Gauss** (1777-1855) Germany

Carl Friedrich Gauss, the "Prince of Mathematics," exhibited his calculative powers when he corrected his father's arithmetic before the age of three. His revolutionary nature was demonstrated at age twelve, when he began questioning the axioms of Euclid. His genius was confirmed at the age of nineteen when he proved that the regular n-gon was constructible, for odd n, if and only if n is the product of distinct prime Fermat numbers. At age 24 he published *Disquisitiones Arithmeticae*, probably the greatest book of pure mathematics ever.

Gauss built the theory of complex numbers into its modern form, including the notion of "monogenic" functions which are now ubiquitous in mathematical physics. The other contributions of Gauss are quite numerous and include the Fundamental Theorem of Algebra (that an n-th degree polynomial has n complex roots), hypergeometric series, foundations of statistics, and differential geometry. Gauss was the premier number theoretician of all time, proving Euler's Law of Quadratic Reciprocity. He also did important work in geometry, providing an improved solution to Apollonius' famous problem of tangent circles; stating and proving the *Fundamental Theorem of Normal Axonometry*, and solving astronomical problems related to comet orbits and navigation by the stars. Gauss also did important work in several areas of physics, and invented the heliotrope.

Much of Gauss's work wasn't published: unbeknownst to his colleagues it was Gauss who first discovered doubly periodic elliptic functions, non-Euclidean geometry, a prime distribution formula, quaternions, foundations of topology, the Law of Least Squares, Dirichlet's class number formula, the butterfly procedure for rapid calculation of Fourier series, and even the rudiments of knot theory. Also in this category is the Fundamental Theorem of Functions of a Complex Variable (that the line-integral over a closed curve of a monogenic function is zero): he proved this first but let Cauchy take the credit. Gauss is widely agreed to be the most brilliant and productive mathematician who ever lived and many would rank him #1; however several of the others on the list had more *historical* importance. Abel hints at a reason for this: "[Gauss] is like the fox, who effaces his tracks in the sand."

Gauss once wrote "It is not knowledge, but the act of learning, ... which grants the greatest enjoyment. When I have clarified and exhausted a subject, then I turn away from it, in order to go into darkness again ..."

Siméon Denis **Poisson** (1781-1840) France

Simeon Poisson was a protégé of Laplace and, like his mentor, is considered to be among the greatest applied mathematicians ever. Poisson was an extremely prolific researcher and also an excellent teacher. In addition to importance advances in several areas of physics, Poisson made important contributions to Fourier analysis, definite integrals, statistics, partial differential equations, calculus of variations and other fields of mathematics.

Jean-Victor **Poncelet** (1788-1867) France

After studying under Monge, Poncelet became an officer in Napoleon's army, then a prisoner of the Russians. To keep up his spirits as a prisoner he devised and solved mathematical problems using charcoal and the walls of his prison cell instead of pencil and paper. During this time he reinvented projective geometry. Regaining his freedom, he wrote many papers, made numerous contributions to geometry; he also made contributions to practical mechanics. Poncelet is considered one of the most influential geometers ever; he is especially noted for his Principle of Continuity, an intuition with broad application. His notion of imaginary solutions in geometry was inspirational. Although projective geometry had been studied earlier by mathematicians like Desargues, Poncelet's work excelled and served as an inspiration for other branches of mathematics including algebra, topology, Cayley's invariant theory and group-theoretic developments by Lie and Klein. His theorems of geometry include his Closure Theorem about Poncelet Traverses and the Poncelet-Brianchon Hyperbola Theorem. Perhaps his most famous theorem, although it was left to Steiner to complete a proof, is the beautiful Poncelet-Steiner Theorem about straight-edge constructions.

Augustin-Louis **Cauchy** (1789-1857) France

Cauchy was extraordinarily prodigious, prolific and inventive. Home-schooled, he awed famous mathematicians at an early age. In contrast to Gauss and Newton, he was almost over-eager to publish; in his day his fame surpassed that of Gauss and has continued to grow. Cauchy did important work in analysis, algebra and number theory. One of his important contributions was the "theory of substitutions" (permutation group theory).

Cauchy's research also included convergence of infinite series, differential equations, determinants, and probability. He invented the calculus of residues. Although he was one of the first great mathematicians to focus on abstract mathematics (another was Euler), he also made important contributions to mathematical physics, e.g. the theory of elasticity. He was the first to prove Fermat's conjecture that every positive integer can be expressed as the sum of k k-gon numbers for any k, and also refined Euler's results in discrete topology. Another of Cauchy's contributions was his insistence on rigorous proofs.

One of the duties of a great mathematician is to nurture his successors, but Cauchy selfishly dropped the ball on both of the two greatest young mathematicians of his day, mislaying the key manuscripts of *both* Abel and Galois. For this historical *miscontribution* I've demoted Cauchy slightly.

Jakob **Steiner** (1796-1863) Switzerland

Jakob Steiner made many major advances in synthetic geometry, hoping that classical methods could avoid any need for analysis; and indeed he was often able to equal or surpass methods of the calculus of variations using just pure geometry. Although the *Principle of Duality* underlying projective geometry was already known, he gave it a radically new and more productive basis, and created a new theory of conics. His work combined generality, creativity and rigor.

Steiner developed several famous construction methods, e.g. for a triangle's smallest circumscribing and largest inscribing ellipses. Among many famous and important theorems of classic and projective geometry, he proved that the Wallace lines of a triangle lie in a 3-pointed hypocycloid, developed a formula for the partitioning of space by planes, and a fact about the surface areas of tetrahedra. Perhaps his three most famous theorems are the Isoperimetric Theorem (among solids of equal volume the sphere will have minimum area, etc.); the Poncelet-Steiner Theorem (lengths constructible with straightedge and compass can be constructed with straightedge alone as long as the picture plane contains the center and circumference of some circle); and his theorem about self-homologous elements in projective geometry. Steiner is often called, along with Apollonius of Perga (who lived 2000 years earlier), one of the two greatest pure geometers ever. (The qualifier "pure" is added to exclude such geniuses as Archimedes, Newton and Pascal from this comparison! Despite Steiner's extreme brilliance and productivity I've omitted him from the Top Sixty list: several geometers had much more historic influence, and as *solely* a geometer he arguably lacked "depth.")

Steiner once wrote: "For all their wealth of content, ... music, mathematics, and chess are resplendently useless (applied mathematics is a higher plumbing, a kind of music for the police band). They are metaphysically trivial, irresponsible. They refuse to relate outward, to take reality for arbiter. This is the source of their witchery."

Niels Henrik **Abel** (1802-1829) Norway

At an early age, Niels Abel studied the works of the greatest mathematicians, found flaws in their proofs, and resolved to reprove some of these theorems rigorously. He was the first to fully prove the general case of Newton's Binomial Theorem, one of the most widely applied theorems in mathematics. Perhaps his most famous achievement was the (deceptively simple) Abel's Theorem of Convergence (published posthumously), one of the most important theorems in analysis; but there are several other Theorems which bear his name. Abel also made contributions in algebraic geometry and the theory of equations.

Inversion (replacing $\mathbf{y = f(x)}$ with $\mathbf{x = f^{-1}(y)}$) is a key idea in mathematics (consider Newton's Fundamental Theorem of Calculus); Abel developed this insight. One of the most respected mathematicians of Abel's day had spent a lifetime studying elliptic integrals, but Abel inverted these to get elliptic functions, which quickly became a productive field of mathematics, and led to more general complex-variable functions, which were important to the development of both abstract and applied mathematics.

Finding the roots of polynomials is a key mathematical problem: the general solution of the quadratic equation was known by ancients; the discovery of general methods for solving polynomials of degree three and four is usually treated as the major math achievement of the 16th century; so for over two centuries an algebraic solution for the general 5th-degree polynomial (quintic) was a Holy Grail sought by most of the greatest mathematicians. Abel proved that most quintics did *not* have such solutions. This discovery, at the age of only nineteen, would have quickly awed the world, but Abel was impoverished, had few contacts, and spoke no German. When Gauss received Abel's manuscript he discarded it unread, assuming the unfamiliar author was just another crackpot trying to square the circle or some such. His genius was too great for him to be ignored long, but, still impoverished, Abel died of tuberculosis at the age of twenty-six. His fame lives on and even the lower-case word 'abelian' is applied to several concepts. Hermite said "Abel has left mathematicians enough to keep them busy for 500 years."

Carl G. J. **Jacobi** (1804-1851) Germany

Jacobi was a prolific mathematician who did decisive work in the algebra and analysis of complex variables, and did work in number theory (e.g. cubic reciprocity) which excited Carl Gauss. He is sometimes described as the successor to Gauss. As an algorist (manipulator of involved algebraic expressions), he may have been surpassed only by Euler and Ramanujan. Jacobi was also an especially inspirational math teacher.

Jacobi's most important early achievement was the theory of elliptic functions. He also made important advances in many other areas, including higher fields, number theory, algebraic geometry, differential equations, theta functions, q-series, determinants, Abelian functions, and physics. He devised the algorithms still used to calculate eigenvectors and for other important matrix manipulations. Jacobi was the first to apply elliptic functions to number theory, producing a new proof of Fermat's famous conjecture (Lagrange's theorem) that every integer is the sum of four squares.

Like Abel, as a young man, Jacobi attempted to factor the general quintic equation. Unlike Abel, he seems never to have considered proving its impossibility. This fact is sometimes cited to show that despite Jacobi's creativity, his ill-fated contemporary was the more brilliant genius.

Johann Peter Gustav Lejeune **Dirichlet** (1805-1859) Germany

Dirichlet was preeminent in algebraic and analytic number theory, but did advanced work in several other fields as well: He discovered the modern definition of function, the Voronoi diagram of geometry, and important concepts in differential equations, topology, and statistics. Although he was one of the foremost mathematicians of the early 19th century, he is often overlooked. (I rank him higher than most Lists of Great Mathematicians do.) Dirichlet's proofs were noted both for great ingenuity and absolute rigor; he was an important teacher, interpreting the work of Gauss and mentoring famous mathematicians like Leopold Kronecker and Ferdinand Eisenstein.

As an impoverished lad Dirichlet spent his money on math textbooks; Gauss' masterwork became his life-long companion. Fermat and Euler had proved the impossibility of $\mathbf{x^k + y^k = z^k}$ for k = 4 and k = 3; Dirichlet became famous by proving impossibility for k = 5 at the age of 20. Later he proved the case k = 14 and, later still, found the flaw in Kummer's proof of the general case (Kummer ignored that unique factorization does not hold for the quadratic fields Dirichlet had invented to address the problem). More important than his work with Fermat's Last Theorem was his Unit Theorem, considered one of the most important theorems of algebraic number theory. The Unit Theorem is unusually difficult to prove; it is said that Dirichlet discovered the proof while listening to music in the Sistine Chapel. A key step in the proof uses "Dirichlet's Pigeonhole Principle", a trivial idea but which Dirichlet applied with great ingenuity.

Dirichlet also did important work in analysis and is considered the founder of analytic number theory. He invented a method of L-series to prove that any arithmetic series has an infinity of primes. It was Dirichlet who proved the fundamental Theorem of Fourier series: that periodic analytic functions can always be represented as a simple trigonometric series. Other fundamental results Dirichlet contributed to analysis and number theory include a theorem about Diophantine approximations, *Dirichlet's Principle* and his *Class Number Formula*.

William Rowan (Sir) **Hamilton** (1805-1865) Ireland

Hamilton was a childhood prodigy. Home-schooled and self-taught, he started as a student of languages and literature, was influenced by an arithmetic prodigy his own age, read Euclid, Newton and Lagrange, found an error by Laplace, and made new discoveries in optics; all this before the age of seventeen when he first attended school! At college he enjoyed unprecedented success in all fields, but his undergraduate days were cut short abruptly by his appointment as

Trinity Professor of Astronomy at the age of 22. He soon began publishing his revolutionary treatises on optics, in which he developed the Principle of Least Action. (Fermat, Leibniz, Maupertuis, and Lagrange also deserve credit for this Principle.) He predicted that some crystals would have an hitherto unknown "conical" refraction mode; this was confirmed experimentally. Hamilton had major early influence in quantum theory with his equations for Least Action, concept of configuration space, and his wave-particle duality which would be further developed by Planck and Einstein.

Hamilton also made revolutionary contributions to dynamics, differential equations, the theory of equations, numerical analysis, fluctuating functions, and graph theory (he marketed a puzzle based on his *Hamiltonian paths*). He invented the ingenious hodograph. In addition to his brilliance and creativity, Hamilton was renowned for thoroughness and produced voluminous writings on several subjects.

Hamilton himself considered his greatest accomplishment to be the development of quaternions, a non-Abelian field to handle 3-D rotations. While there is no 3-D analog to the Gaussian complex-number plane (based on the equation $\boldsymbol{i^2 = -1}$), quaternions derive from a 4-D analog based on $\boldsymbol{i^2 = j^2 = k^2 = ijk = -1}$. (Despite their being "obsoleted" by more general matrix and tensor methods, quaternions are still in wide engineering use because of certain practical advantages.)

Hamilton once wrote: "On earth there is nothing great but man; in man there is nothing great but mind."

Évariste **Galois** (1811-1832) France

Galois, who died before the age of twenty-one, not only never became a professor, but was barely allowed to study as an undergraduate. His output of papers, mostly published posthumously, is much smaller than most of the others on this list, yet it is considered among the most awesome works in mathematics. He applied group theory to the theory of equations, revolutionizing both fields. (Galois coined the mathematical term "group.") While Abel was the first to prove that some polynomial equations had no algebraic solutions, Galois established the necessary and sufficient condition for algebraic solutions to exist. His principle treatise was a letter he wrote the night before his fatal duel, of which Hermann Weyl wrote: "This letter, if judged by the novelty and profundity of ideas it contains, is perhaps the most substantial piece of writing in the whole literature of mankind."

Galois' last words (spoken to his brother) were "Ne pleure pas, Alfred! J'ai besoin de tout mon courage pour mourir à vingt ans!" This tormented life, with its pointless early end, is one of the great tragedies of mathematical history.

James Joseph **Sylvester** (1814-1897) England, U.S.A.

Sylvester made important contributions to several fields, including matrix theory (he coined the term "matrix"), invariant theory, number theory, partition theory, reciprocant theory, and combinatorics. He was also a linguist, a poet, and did work in mechanics and optics. Sylvester once wrote, "May not music be described as the mathematics of the sense, mathematics as music of the reason?"

Karl Wilhelm Theodor **Weierstrass** (1815-1897) Germany

Weierstrass devised new definitions for the primitives of calculus and was then able to prove several fundamental but hitherto unproven theorems. He developed new insights in several fields including the calculus of variations and trigonometry. Weierstrass shocked his colleagues when he demonstrated a continuous function which is differentiable nowhere. He found simpler proofs of many existing theorems, including Gauss' Fundamental Theorem of Algebra and the fundamental Hermite-Lindemann Transcendence Theorem. He found a fundamental flaw in Steiner's proof of the Isoperimetric Theorem, and became the first to supply a fully rigorous proof of that famous and ancient result. Starting strictly from the integers, he also applied his axiomatic methods to a definition of irrational numbers.

Weierstrass demonstrated extreme brilliance as a youth, but during his college years he detoured into drinking and dueling and ended up as a degreeless secondary school teacher. During this time he studied Abel's papers, developed results in elliptic and Abelian functions, and independently proved the Fundamental Theorem of Functions of a Complex Variable. He was interested in power series and felt that others had overlooked the importance of Abel's Theorem. Eventually one of his papers was published in a journal; he was immediately given an honorary doctorate and was soon regarded as one of the best and most inspirational mathematicians in the world. (In 1873 Hermite called him "the Master of all of us." Bell called him "probably the greatest mathematical teacher of all time.") Weierstrass is now called the "Father of Modern Analysis."

Weierstrass once wrote: "A mathematician who is not also something of a poet will never be a complete mathematician."

George **Boole** (1815-1864) England

George Boole was a precocious child who impressed by teaching himself classical languages, but was too poor to attend college and became an elementary school teacher at age 16. He gradually developed his math skills; as a young man he published a paper on the calculus of variations, and soon became one of the most respected mathematicians in England despite having no formal training. He was noted for work in symbolic logic, algebra and analysis, and also was apparently the first to discover invariant theory. When he followed up Augustus de Morgan's earlier work in symbolic logic, de Morgan insisted that Boole was the true master of that field, and begged his friend to finally study mathematics at university. Boole couldn't afford to, and had to be appointed Professor instead!

Although very few recognized its importance at the time, it is Boole's work in Boolean algebra and symbolic logic for which he is now remembered; this work inspired computer scientists like Claude Shannon. Boole's book *An Investigation of the Laws of Thought* prompted Bertrand Russell to label him the "discoverer of pure mathematics."

Boole once said "No matter how correct a mathematical theorem may appear to be, one ought never to be satisfied that there was not something imperfect about it until it also gives the impression of being beautiful."

Pafnuti Lvovich **Chebyshev** (1821-1894) Russia

Pafnuti Chebyshev (or Pafnuty Tschebyscheff) was noted for work in probability, number theory, approximation theory, integrals, the theory of equations, and applied mathematics. His famous theorems include a new version of the Law of Large Numbers, an important result in integration of radicals first conjectured by Abel, an incomplete form of the Prime Number Theorem, and the fact that there is always a prime between any *n* and *2n*. He introduced new concepts such as orthogonal polynomials, and did significant work on the zeta function before Riemann did. Several concepts are named after him, including Chebyshev polynomials. He invented the Chebyshev linkage, a mechanical device to convert rotational motion to straight-line motion.

Arthur **Cayley** (1821-1895) England

Cayley was the third most prolific mathematician in history, behind only Euler and Erdos. A list of the branches of mathematics Cayley pioneered will seem like an exaggeration: he was the essential founder of modern group theory, matrix algebra, and higher dimensional geometry, as well as the theory of invariants. He stated and proved the Cayley-Hamilton Theorem. He also did original research in combinatorics, elliptic and Abelian functions, and projective geometry. One of his many famous geometric theorems is a generalization of Pascal's Mystic Hexagram result; another resulted in an elegant proof of the Quadratic Reciprocity law.

Cayley may have been the *least* eccentric of the great mathematicians: In addition to his life-long love of mathematics, he enjoyed hiking, painting, reading fiction, and had a happy married life. He worked as a lawyer for many years, then as professor, and finished his career in the limelight as President of the British Association for the Advancement of Science. He and the great mathematician James Joseph Sylvester (1814-1897) were a source of inspiration to each other. These two, along with Charles Hermite, are considered the founders of the important theory of invariants. Though applied first to algebra, the notion of invariants is useful in many areas of mathematics.

Cayley once wrote: "As for everything else, so for a mathematical theory: beauty can be perceived but not explained."

Charles **Hermite** (1822-1901) France

Hermite studied the works of Lagrange and Gauss from an early age and soon developed an alternate proof of Abel's famous quintic impossibility result. He attended the same college as Galois and also had trouble passing their examinations, but soon became highly respected by Europe's greatest mathematicians for his successes in number theory and elliptic functions. Along with Cayley and Sylvester, he founded the important theory of invariants. He was a kindly modest man and an inspirational teacher. Among his students was Poincare, who said of Hermite, "He never evokes a concrete image, yet you soon perceive that the more abstract entities are to him like living creatures."

Although he and Abel had proved that the general quintic lacked algebraic solutions, Hermite introduced an elliptic analog to the circular trigonometric functions and used these to provide a general solution for the quintic equation. He developed novel ways to apply analysis to number theory. He developed the concept of complex conjugate which is now ubiquitous in mathematical physics and matrix theory. He was first to prove that the Stirling and Euler generalizations of the factorial function are equivalent. Hermite's most famous result was his ingenious proof that *e* (along with a broad class of related numbers) is transcendental.

Ferdinand Gotthold Max **Eisenstein** (1823-1852) Germany

Eisenstein was born into severe poverty and suffered health problems throughout his short life, but was still one of the more significant mathematicians of his era. Today's mathematicians who study Eisenstein are invariably amazed by his brilliance and originality. He made revolutionary advances in number theory, algebra and analysis, and was also a composer of music. He anticipated ring theory, developed a new basis for elliptic functions, proved several theorems about cubic and quartic reciprocity, and much more.

Eisenstein was a young prodigy; he once wrote "As a boy of six I could understand the proof of a mathematical theorem more readily than that meat had to be cut with one's knife, not one's fork." Despite his early death, he is considered one of the greatest number theorists ever. Gauss named Eisenstein, along with Newton and Archimedes, as one of the three epoch-making mathematicians of history.

Georg Friedrich Bernhard **Riemann** (1826-1866) Germany

Riemann was a phenomenal genius whose work was exceptionally deep, creative and rigorous; he made revolutionary contributions in many areas of pure mathematics, and also inspired the development of physics. He had poor physical health and died at an early age, yet is still considered to be among the most productive mathematicians ever. He was the master of complex analysis. He applied topology to analysis, and analysis to number theory, making revolutionary contributions to all three fields. He introduced the clarifying notion of the Riemann integral. Riemann's masterpieces include differential geometry, tensor analysis, non-Euclidean geometry, the theory of functions, and, especially, the theory of manifolds. He generalized the notions of distance and curvature and, therefore, described new possibilities for the geometry of space itself. Like his mathematical peers (Gauss, Archimedes, Newton), Riemann was intensely interested in physics. His theory unifying electricity, magnetism and light was supplanted by Maxwell's theory; however modern physics, beginning with Einstein's relativity, relies on Riemann's notions of the geometry of space.

Riemann's teacher was Carl Gauss, who helped steer the young genius towards pure mathematics. Gauss selected "On the hypotheses that Lie at the Foundations of Geometry" as Riemann's first lecture; with this famous lecture Riemann advanced Gauss' initial effort in differential geometry, extended it to multiple dimensions, and introduced the new and important theory of differential manifolds. Five years later, to celebrate his election to the Berlin Academy, Riemann presented a lecture "On the Number of Prime Numbers Less Than a Given Quantity," for which "Number" he presented and proved an *exact* formula, albeit weirdly complicated and seemingly intractable. Numerous papers have been written on the distribution of primes, but Riemann's contribution is incomparable, despite that his Berlin Academy lecture was his only paper ever on the topic, and number theory was far from his specialty. In the lecture he posed the "Hypothesis of Riemann's zeta function," (**ζ()** was defined in Euler's mini-bio) which is now considered the most important and famous unsolved problem in mathematics. (Asked what he would first do, if he were magically awakened after centuries, David Hilbert replied "I would ask whether anyone had proved the Riemann Hypothesis." The Riemann Hypothesis "simply" states that in all solutions of $\zeta(s = a+bi) = 0$, either **s** has real part **a=1/2** or imaginary part **b=0**.)

Despite his great creativity (Gauss praised Riemann's "gloriously fertile originality"), Riemann once said: "If only I had the theorems! Then I should find the proofs easily enough."

Julius Wilhelm Richard **Dedekind** (1831-1916) Germany

Dedekind was one of the most innovative mathematicians ever; his clear expositions and rigorous axiomatic methods had great influence. He made seminal contributions to abstract algebra and algebraic number theory as well as mathematical foundations. He was one of the first to pursue Galois Theory, making major advances there and pioneering in the application of group theory to other branches of mathematics. Dedekind also invented a system of fundamental axioms for arithmetic, worked in probability theory and complex analysis, and invented prime partitions and modular lattices. Dedekind may be most famous for his theory of ideals and rings; Kronecker and Kummer had begun thus, but Dedekind gave it a more abstract and productive basis, which was developed further by Hilbert, Noether and Weil.

Dedekind was concerned with rigor, writing "nothing capable of proof ought to be accepted without proof." Before him, the real numbers, continuity, and infinity all lacked rigorous definitions. The axioms Dedekind invented allow the integers and rational numbers to be built and his "Dedekind Cut" then led to a rigorous and useful definition of the real numbers. Dedekind anticipated and inspired Cantor's work: he introduced the notion that a bijection implied equinumerosity, used this to define infinitude (a set is infinite if equinumerous with its proper subset), and proved the Cantor-Bernstein Theorem; he should thus be considered a co-inventor of Cantor's set theory.

Georg **Cantor** (1845-1918) Russia, Germany

Cantor created modern set theory, defining cardinal numbers, well-ordering, ordinal numbers, and discovering the Theory of Transfinite Numbers. He defined equality between cardinal numbers based on the existence of a bijection, and was the first to demonstrate that the real numbers have a higher cardinal number than the integers. (The rationals have the same cardinality as the integers; the reals have the same cardinality as the points of N-space.) Although there are infinitely many distinct transfinite numbers, Cantor conjectured that *C*, the cardinality of the reals, was the second smallest transfinite number. This "Continuum Hypothesis" was included in Hilbert's famous List of Problems, and was partly resolved many years later: Cantor's Continuum Hypothesis is an "Undecidable Statement" of Set Theory.

Cantor's revolutionary set theory attracted vehement opposition from Poincare ("grave disease"), Kronecker (Cantor was a "charlatan" and "corrupter of youth"), Wittgenstein ("laughable nonsense"), and even theologians. David Hilbert had kinder words for it: "The finest product of mathematical genius and one of the supreme achievements of purely intellectual human activity." Cantor's own attitude was expressed with "The essence of mathematics lies in its freedom." Cantor's invention of modern set theory is now considered one of the most important and creative achievements in modern mathematics.

Cantor also made advances in number theory and trigonometric series. He gave the modern definition of irrational numbers, and anticipated the theory of fractals. Cantor once wrote "In mathematics the art of proposing a question must be held of higher value than solving it."

Christian Felix **Klein** (1849-1925) Germany

Klein's key contribution was an application of invariant theory to unify geometry with group theory. This radical new view of geometry inspired Sophus Lie's Lie groups, and also led to the remarkable unification of Euclidean and non-Euclidean geometries which is probably Klein's most famous result. Klein did other work in function theory, providing links between several areas of mathematics including number theory, group theory, hyperbolic geometry, and abstract algebra. His Klein's Quartic curve and popularly-famous "Klein's bottle" were among several useful results from his new approaches to groups and higher-dimensional geometries and equations. Klein did important work in mathematical physics, e.g. writing about gyroscopes. He facilitated David Hilbert's early career, publishing his controversial Finiteness Theorem and declaring it "without doubt the most important work on general algebra [the leading German journal] ever published."

Klein is also famous for his book on the icosahedron, reasoning from its symmetries to develop the elliptic modular and automorphic functions which he used to solve the general quintic equation. He formulated a "grand uniformization theorem" about automorphic functions but suffered a health collapse before completing the proof. His focus then changed to teaching; he devised a mathematics curriculum for secondary schools which had world-wide influence. Klein once wrote "... mathematics has been most advanced by those who distinguished themselves by intuition rather than by rigorous proofs."

Jules Henri **Poincaré** (1854-1912) France

Poincaré was clumsy and frail and supposedly flunked an IQ test, but he was one of the most creative mathematicians ever, and surely the greatest mathematician of the Constructivist ("intuitionist") style. Poincare is sometimes called the Father of Topology (a title also used for Euler and Brouwer), but produced a large amount of brilliant work in many other areas of mathematics, and also found time to become a famous popular writer of philosophy, saying, for example, "Mathematics is the art of giving the same name to different things."

Poincare's masterpieces include combinatorial (or algebraic) topology, the theory of differential equations, foundations of homology, the theory of periodic orbits, and the discovery of automorphic functions (a unifying foundation for the trigonometric and elliptic functions). He anticipated modern chaos theory. He posed "Poincare's conjecture," which for an entire century was one of the most famous unsolved problems in mathematics and which can be explained without equations to a layman (provided the layman can visualize 3-D surfaces in 4-space). Recently Poincare's conjecture was settled and the first Million Dollar math prize in history is likely to be awarded.

As with all the greatest mathematicians, Poincaré was interested in physics. He made revolutionary advances in fluid dynamics and celestial motions. He is sometimes considered to be a co-inventor of the Special Theory of Relativity. With his fame, he helped the world recognize the importance of the new physical theories of Einstein and Planck.

David **Hilbert** (1862-1943) Prussia, Germany

Hilbert was preeminent in many fields of mathematics, including axiomatic theory, invariant theory, algebraic number theory, class field theory and functional analysis. His examination of calculus led him to the invention of "Hilbert space," considered one of the key concepts of functional analysis and modern mathematical physics. He was a founder of fields like metamathematics and modern logic. He was also the founder of the "Formalist" school which opposed the "Intuitionism" of Kronecker and Brouwer. He developed a new system of definitions and axioms for geometry, replacing the 2200 year-old system of Euclid. As a young Professor he proved his "Finiteness Theorem," now regarded as one of the most important results of general algebra. The methods he used were so novel that, at first, the "Finiteness Theorem" was rejected for publication as being "theology" rather than mathematics! In number theory, he proved Waring's famous conjecture which is now known as the Hilbert-Waring theorem.

Any one man can only do so much, so the greatest mathematicians should help nurture their colleagues. Hilbert provided a famous List of 23 Unsolved Problems, which inspired and directed the development of 20th-century mathematics. Hilbert was warmly regarded by his colleagues and students, and contributed to the careers of several great mathematicians and physicists including Georg Cantor, Hermann Minkowski, Hermann Weyl, John von Neumann, Emmy Noether, Alonzo Church, and Albert Einstein.

Eventually Hilbert turned to physics and made key contributions to classical and quantum physics and to general relativity. (Hilbert was a modest man: some historians believe the "Einstein Field Equations" should carry Hilbert's name.)

Hermann **Minkowski** (1864-1909) Lithuania, Germany

Minkowski won a prestigious prize at age 18 for reconstructing a lost proof of Eisenstein. He worked with quadratic forms and continued fractions, which eventually led him to the new methods called Geometric Number Theory. He became a close friend of David Hilbert and one of Einstein's teachers. Minkowski is particularly famous for inventing *Minkowski space* to deal with Einstein's Special Theory of Relativity. (Poincare should also get partial credit for this.)

Jacques Salomon **Hadamard** (1865-1963) France

Hadamard made revolutionary advances in several different areas of mathematics, especially complex analysis, analytic number theory, differential geometry, partial differential equations, symbolic dynamics, and matrix theory; for this reason he is sometimes considered the "Last Universal Mathematician." He also made contributions to physics. One of the most famous results in mathematics is the Prime Number Theorem, that there are approximately **n/log n** primes less than **n**. This result was conjectured by Legendre and Gauss, attacked cleverly by Riemann and Chebyshev, and finally proved by Hadamard. (Valee-Poussin proved it also, but Hadamard's proof is considered more elegant and useful.) Several other important theorems in various fields are named after Hadamard, especially Hadamard's Inequality of Determinants; another important achievement was his survey of Poincare's work. Hadamard was also an influential teacher, Andre Weil and others acknowledging him as key inspiration.

Godfrey Harold **Hardy** (1877-1947) England

Hardy was an extremely prolific research mathematician who did important work in analysis (especially the theory of integration), number theory, global analysis, and analytic number theory. He was also an excellent teacher and wrote several excellent textbooks, as well as a famous treatise on the mathematical mind. He wrote "I am interested in mathematics only as a creative art." Although he emphasized pure mathematics (actually abhorring applied mathematics), his work has found application in population genetics, cryptography, thermodynamics and particle physics.

Hardy is especially famous (and important) for his encouragement of and collaboration with Ramanujan. Among many results of this collaboration was the Hardy-Ramanujan Formula for partition enumeration, which Hardy later used as a model to develop the Hardy-Littlewood Circle Method; Hardy first used this method to prove stronger versions of the Hilbert-Waring theorem, and in prime number theory; the method has continued to be a very productive tool in analytic number theory.

Albert **Einstein** (1879-1955) Germany, Switzerland, U.S.A.

Albert Einstein was probably the greatest physicist in all of history (some would say Isaac Newton shares that honor). The atomic theory achieved general acceptance only after Einstein's 1905 paper which showed that atoms' discreteness explained Brownian motion. Another famous 1905 paper introduced the famous equation $\mathbf{E = mc^2}$; yet Einstein published other papers that same year, two of which were more important and influential than either of the two just mentioned! No wonder that physicists speak of the *Miracle Year* without bothering to qualify it as *Einstein's Miracle Year*! (Altogether Einstein published at least 300 books or papers on physics.)

Although most famous for his Special and General Theories of Relativity, Einstein was also an early pioneer in quantum theory (although he disagreed with the usual view of the Uncertainty Principle). He received the Nobel Prize for discovering the photoelectric effect, but never received a Nobel Prize for either Theory of Relativity even though these were two of the most creative and important scientific theories ever.

Einstein certainly has the breadth, depth, and historical importance to qualify for this list; but his genius and significance were not in the field of pure mathematics. He *was* a mathematician, however, and I've chosen to include him on this list for the same reason I include Kepler: his extreme greatness overrides his focus away from math. Einstein ranks #10 on Michael Hart's famous list of the Most Influential Persons in History. Einstein once wrote "... the creative principle resides in mathematics [; thus] I hold it true that pure thought can grasp reality, as the ancients dreamed."

Luitzen Egbertus Jan **Brouwer** (1881-1966) Holland

Brouwer is often considered the Father of Topology; his two most important theorems were the Fixed Point Theorem, and the Invariance of Dimension. He developed the method of simplicial approximations, important to algebraic topology; he also did work in geometry, set theory, measure theory, complex analysis and the foundations of mathematics.

Brouwer is most famous as the founder of Intuitionism, a philosophy of mathematics in sharp contrast to Hilbert's Formalism, but Brouwer's philosophy also involved ethics and aesthetics and has been compared with those of Schopenhauer and Nietzsche. Part of his mathematics thesis was rejected as "... interwoven with some kind of pessimism and mystical attitude to life which is not mathematics ..." As a young man, Brouwer spent a few years to develop topology, but once his great talent was demonstrated and he was offered prestigious professorships, he devoted himself to Intuitionism, and acquired a reputation as eccentric and self-righteous.

Intuitionism has had a significant influence, although few strict adherents. Since only constructive proofs are permitted, strict adherence would slow mathematical work. This didn't worry Brouwer who once wrote: "The construction itself is an art, its application to the world an evil parasite."

Amalie Emma **Noether** (1882-1935) Germany

Noether was an innovative researcher who made several major advances in abstract algebra, including a new theory of ideals, the inverse Galois problem, and the general theory of commutative rings. She originated novel reasoning methods, especially one based on "chain conditions," which advanced invariant theory and abstract algebra; her insistence on generalization led to a unified theory of modules and Noetherian rings; some of her work anticipated modern category theory. Her invention of homology groups revolutionized topology.

Noether also made advances in mathematical physics; Noether's Theorem establishing that certain symmetries imply conservation laws has been called the most important Theorem in physics since the Pythagorean Theorem. Noether was an unusual and inspiring teacher; and generously helped students and colleagues, even allowing them to claim her work as their own. Noether was close friends with the other greatest mathematicians of her generation: Hilbert, von Neumann, and Weyl. Weyl once said he was embarrassed to accept the famous Professorship at Göttingen because Noether was his "superior as a mathematician." Many would agree that Emmy Noether was the greatest female mathematician ever.

Hermann Klaus Hugo **Weyl** (1885-1955) Germany, U.S.A.

Weyl studied under Hilbert and became one of the premier mathematicians of the 20th century. He excelled at many fields including integral equations, harmonic analysis, analytic number theory, and the foundations of mathematics, but he is most respected for his revolutionary advances in geometric function theory (e.g., differentiable manifolds), the theory of compact groups (incl. representation theory), and theoretical physics (e.g., Weyl tensor, gauge field theory and invariance). For a while, Weyl was a disciple of Brouwer's Intuitionism and helped advance that doctrine, but he eventually found it too restrictive. Weyl was also a very influential figure in all three major fields of 20th century physics: relativity, unified field theory and quantum mechanics.

Weyl once wrote: "My work always tried to unite the Truth with the Beautiful, but when I had to choose one or the other, I usually chose the Beautiful."

Srinivasa **Ramanujan** Iyengar (1887-1920) India

Like Abel, Ramanujan was a self-taught prodigy who lived in a country distant from his mathematical peers, and suffered from poverty: childhood dysentery and vitamin deficiencies probably led to his early death. Yet he produced 4000 theorems or conjectures in number theory, algebra, and combinatorics. His specialties included infinite series, elliptic functions, continued fractions, partition enumeration, definite integrals, modular equations, gamma functions, "mock theta" functions, hypergeometric series, and "highly composite" numbers. Much of his methodology, including unusual ideas about divergent series, was his own invention. (As a young man he made the absurd claim that **1+2+3+4+... = -1/12**. Later it was noticed that this claim translates to a true statement about the Riemann zeta function, with which Ramanujan was unfamiliar.) Ramanujan's innate ability for algebraic manipulations equaled or surpassed that of Euler and Jacobi. Many of Ramanujan's results would probably never have been discovered without him, and are so inspirational that there is a periodical dedicated to them. The theories of strings and crystals have benefited from Ramanujan's work. (Today some professors achieve fame just by finding a new proof for one of Ramanujan's many results.) Unlike Abel, who insisted on rigorous proofs, Ramanujan often omitted proofs. (Ramanujan may have had unrecorded proofs, poverty leading him to use chalk and erasable slate rather than paper.) Unlike Abel, much of whose work depended on the complex numbers, most of Ramanujan's work focused on real numbers. Despite these limitations, Ramanujan is considered one of the greatest geniuses ever.

Because of its fast convergence, an odd-looking formula of Ramanujan is often used to calculate **π**:

$$99^2 / \pi = \sqrt{8} \sum_{k=0,\infty} (4k! (1103+26390 k) / (k!^4 396^{4k}))$$

Stefan **Banach** (1892-1945) Poland

Stefan Banach was a self-taught mathematician who is most noted as the Founder of Functional Analysis and for his contributions to measure theory. Among several important theorems bearing his name are the Uniform Boundedness (Banach-Steinhaus) Theorem, the Open Mapping (Banach-Schauder) Theorem, the Contraction Mapping (Banach fixed-point) Theorem, and the Hahn-Banach theorem. Many of these theorems are of practical value to modern physics; however he also proved the paradoxical Banach-Tarski Theorem, which demonstrates a sphere being rearranged into *two* spheres of the same original size! (Banach's proof uses the Axiom of Choice and is sometimes cited as evidence that that Axiom is false!) The wide range of Banach's work is indicated by the Banach-Mazur results in game theory (which also challenge the axiom of choice). Banach also made brilliant contributions to probability theory, set theory, analysis and topology.

Banach once said "Mathematics is the most beautiful and most powerful creation of the human spirit."

John **von Neumann** (1903-1957) Hungary, U.S.A.

John von Neuman (born Neumann Janos Lajos) was a childhood prodigy who could do very complicated mental arithmetic at an early age. As an adult he was noted for hedonism and reckless driving but also became one of the most prolific geniuses in history, making major contributions to a large variety of branches of mathematics, as well as to quantum physics, economics and computer science.

Von Neumann pioneered the use of models in set theory, thus improving the axiomatic basis of mathematics; he developed von Neumann Algebras; he invented and developed game theory; he invented cellular automata, famously constructing a self-reproducing automaton; he invented elegant definitions for the counting numbers (**0 = {}**, **n+1 = n ∪ {n}**). He also worked in analysis, operator theory, matrix theory, numerical analysis, ergodic theory, continuous geometry, statistics and topology. Von Neumann discovered an ingenious area-conservation paradox related to the famous Banach-Tarski volume-conservation paradox. He inspired some of Godel's famous work (and independently proved Godel's Second Theorem). He is credited with (partial) solution to Hilbert's 5th Problem.

Von Neumann did very important work in fields other than mathematics. By treating the universe as a very-high dimensional phase space, he constructed an elegant mathematical basis (now called von Neumann algebras) for the principles of quantum physics. He advanced philosophical questions about time and logic in modern physics. He played a key role in the design of conventional, nuclear and thermonuclear bombs. He applied game theory and Brouwer's fixed-point theorem to economics, becoming a major figure in that field. His contributions to computer science are many: in addition to co-inventing the stored-program computer, he was first to use pseudo-random number generation, finite element analysis, the merge-sort algorithm, and a "biased coin" algorithm. By implementing wide-number software he joined several other great mathematicians (Archimedes, Apollonius, Liu Hui, Hipparchus, Madhava, Ramanujan) in producing the best approximation to π of his time. At the time of his death, von Neumann was working on a theory of the human brain.

Andrey Nikolaevich **Kolmogorov** (1903-1987) Russia

Kolmogorov had a powerful intellect and excelled in many fields. As a youth he dazzled his teachers by constructing toys that appeared to be "Perpetual Motion Machines." At the age of 19, he achieved fame by finding a Fourier series that diverges almost everywhere, and decided to devote himself to mathematics. He is considered the founder of the fields of intuitionistic logic, algorithmic complexity theory, and modern probability theory. He also excelled in topology, set theory, trigonometric series, and random processes. He (and his student) resolved Hilbert's 13th Problem. While Kolmogorov's work in probability theory had direct applications to physics, Kolmogorov also did work in physics directly, especially the study of turbulence. There are dozens of theorems or equations named after Kolmogorov, such as the "Kolmogorov backward equation" and the intriguing Zero-One Law of "tail events" among random variables.

Kurt **Gödel** (1906-1978) Germany, U.S.A.

Godel, who had the nickname *Herr Warum* ("Mr. Why") as a child, was perhaps the foremost logic theorist ever, clarifying the relationships between various modes of logic. He partially resolved both Hilbert's 1st and 2nd Problems, the latter with a proof so remarkable that it was connected to the drawings of Escher and music of Bach in the title of a famous book. He was a close friend of Albert Einstein, and was first to discover "paradoxical" solutions (e.g. time travel) to Einstein's equations. About his friend, Einstein later said that he had remained at Princeton's Institute for Advanced Study merely "to have the privilege of walking home with Godel." (Like a few of the other greatest 20th-century mathematicians, Godel was very eccentric.)

Two of the major questions confronting mathematics are: (1) are its axioms consistent (its theorems all being true statements)?, and (2) are its axioms complete (its true statements all being theorems)? Godel turned his attention to these fundamental questions. He proved that first-order logic was indeed *complete*, but that the more powerful axiom systems needed for arithmetic (constructible set theory) were necessarily *incomplete*. He also proved that the Axioms of Choice (AC) and the Generalized Continuum Hypothesis (GCH) were *consistent* with set theory, but that set theory's own consistency could not be proven. He may have established that the truths of AC and GCH were *independent* of the usual set theory axioms, but the proof was left to his disciple Paul Cohen.

In Godel's famous proof of Incompleteness, he exhibits a true statement (G) which cannot be proven, to wit "*G (this statement itself) cannot be proven.*" If G could be proven it would be a contradictory true statement, so consistency dictates that it indeed *cannot* be proven. But that's what G says, so G is true! This sounds like mere word play, but building from ordinary logic and arithmetic Godel was able to construct statement G rigorously.

André **Weil** (1906-1998) France, U.S.A.

Weil made profound contributions to many areas of mathematics, especially algebraic geometry, which he connected with number theory. His "Weil conjectures" were very influential; these and other works laid the groundwork for many of Grothendieck's achievements. Weil proved a special case of the Riemann hypothesis; he contributed, at least indirectly, to the recent proof of "Fermat's last Theorem;" he also worked in group theory, general and algebraic topology, differential geometry, sheaf theory, representation theory, and theta functions. He invented several new concepts including vector bundles, and uniform space. His work has found applications in particle physics and string theory. He is considered to be one of the most influential of modern mathematicians.

Weil's biography is interesting. He studied Sanskrit as a child, loved to travel, taught at a Muslim university in India for two years (intending to teach French civilization), wrote as a young man under the famous pseudonym Nicolas Bourbaki, spent time in prison during World War II as a Jewish objector, was almost executed as a spy, escaped to America, and eventually joined Princeton's Institute for Advanced Studies. He once wrote: "Every mathematician worthy of the name has experienced [a] lucid exaltation in which one thought succeeds another as if miraculously."

Paul **Erdös** (1913-1996) Hungary, U.S.A., Israel, etc.

Erdos was a childhood prodigy who became a famous (and famously eccentric) mathematician. He is best known for work in Ramsey Theory, but made contributions in many other fields of mathematics, including graph theory, analytic number theory, probabilistic methods, approximation theory, and combinatorics. He is regarded as the second most prolific mathematician in history, behind only Euler. Although he is widely regarded as an important and influential mathematician, Erdos founded no new field of mathematics: He was a "problem solver" rather than a "theory developer." He's left us several still-unproven intriguing conjectures, e.g. that $\mathbf{4/n = 1/x + 1/y + 1/z}$ has positive-integer solutions for any $\mathbf{n}$.

Erdos liked to speak of "God's Book of Proofs" and discovered new, more elegant, proofs of several existing theorems, including the two most famous and important about prime numbers: Chebyshev's Theorem that there is always a prime between any *n* and *2n*, and Hadamard's Prime Number Theorem itself. He also proved many new theorems, such as the Erdos-Szekeres Theorem about monotone subsequences with its elegant (if trivial) pigeonhole-principle proof.

Alexander **Grothendieck** (1928-) Germany, France

Grothendieck has done brilliant work in several areas of mathematics including number theory, geometry, topology, functional (and topological) analysis, but especially in the fields of algebraic geometry and category theory, both of which he revolutionized. Several mathematical concepts are named after him. Grothendieck is considered a master of abstraction, rigor and presentation. He has produced many important and deep results in homological algebra, most notably his etale cohomology. He developed the theory of sheafs, invented the theory of schemes, and much more. He is most famous for his methods to unify different branches of mathematics, for example using algebraic geometry in number theory.

Grothendieck's radical political philosophy led him to retire from public life while still in his prime, but he is still considered one of the most brilliant mathematicians ever.

Go to a discussion of this list.

Go back to my home page.

Send me some e-mail.
